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## OBSERVATIONS OF THE RADIAL VELOCITIES OF THIRTY-ONE STARS MADE AT THE EMERSON McMILLIN OBSERVATORY

By H. C. LORD

For the past ten years observations of stellar motions in the line of sight have been made at the Emerson McMillin Observatory. Though not all the stars whose velocities it is possible to measure with this instrument have been observed, yet, in view of the optical giants at work in this branch of research today, and in further consideration of the fact that our sky is far from favorable for such work and that it is yearly getting worse, it seems advisable to draw the work to a close and turn to some other line of investigation better suited to our conditions. It seems proper at this time to publish the results thus far secured which have not as yet been given to the public, and to supplement them with a full description of the methods employed.

The instrument used is fully described in this *Journal*.[1] Certain changes have since been made, a description of which may be of general interest. In 1898 the results of a number of observations on

[1] *Astrophysical Journal*, **4**, 50, 1896.

five stars were published. Those plates were all measured by the old Potsdam method of placing the star negative in contact, film to film, with a negative of the solar spectrum and measuring through the glass. Hydrogen alone was used as a source for the comparison spectrum. Shortly after these results were published, Campbell's classic paper, "The Mills Spectrograph of the Lick Observatory,"[1] appeared. Thereafter iron as well as hydrogen was always used as a source for the comparison spectrum. The method of reducing the plates is quite different from Campbell's, and an account of it will be given later.

The spectrograph as originally constructed was provided with two very dense flint prisms. These were so dense that the polished faces rapidly rusted and the spectrograms were fearfully unbalanced, being always underexposed on one side of $H\gamma$, and overexposed on the other. I was convinced that this defect could be largely reduced by the use of lighter glass. In order, however, to secure equal dispersion, it would be necessary to replace the two dense flints by three light ones; and it was a nice question as to how much the gain by the increased transparency of the glass would be counterbalanced by the increased loss by reflection. This can be easily predetermined, provided only we know the coefficient of absorption of the glass for the region of the spectrum covered. Such data are almost entirely wanting, so that it was necessary to measure this coefficient, not only for samples of glass kindly furnished me by Mr. Brashear, but also for the prisms of the old battery of dense flints. The observatory has a single dense-flint prism which, I believe, though I am not certain, was cut from the same block as the prisms of the battery. After many failures, I succeeded in giving the top and bottom faces of this prism a very fair degree of polish.

As this observatory has no apparatus for measurements of this kind, I was compelled to design and build one. The instrument finally used consisted of a spectrometer, the objective of whose collimator had been sawed in two like the objective of a heliometer, the two halves being separated. The slit was provided with a double set of jaws at right angles to one another; one, placed parallel to the refracting edge of the prism; the other, placed immediately in

[1] *Ibid.*, **8**, 123, 1898.

front of the first, serving to fix the width of the spectrum to a nicety. Upon looking into such an instrument, two spectra, illuminated by the same slit, will be seen side by side. The distance apart of the centers of these two spectra depends upon the separation of the two halves of the collimator objective, and obviously their edges may be made to touch simply by changing the length of the slit. A small circular diaphragm was placed in the focal plane of the observing telescope. Upon looking into the eyepiece, a circle of practically monochromatic light appeared, whose upper half was illuminated by the light which passed through the upper half of the collimator objective, and whose lower half by the light which passed through the lower half of the collimator objective. The approximate wavelength of this light could be taken from the graduated circle of the spectrometer. If the areas of the two halves of the objective were equal, the two halves of this colored circle of the light would be equally brilliant and the line of demarcation would disappear. The collimator objective was provided with a cap having two rectangular apertures, the upper one of constant area and the lower one of the same width, but of a length which could be varied with a micrometer screw. The reading of this screw gave at once the relative areas of the two openings.

The sample of glass to be tested was placed on a support immediately in front of the upper opening, and the lower one was closed until the circle appeared uniformly illuminated. The amount of light transmitted was evidently the ratio of the areas of the two openings. No account was taken of the loss by reflection at normal incidence. Table I gives the values of the coefficient of absorption for 100 mm for the four kinds of glass tested. Each determination is the mean of five made at different times. Though the separate results agree fairly well, yet I suspect large constant errors due to causes to be explained later; the relative values are, I believe, fairly accurate. The chief difficulty was in securing uniform illumination of the collimator objective. This was increased as I used an ordinary single achromatic photographic lens of rather short focus; but even this would have been much less troublesome had I had a heliostat whose reflecting surfaces were optical flats. I finally illuminated the slit with the diffused light reflected from a piece of white paper.

The Fraunhofer lines caused no trouble, as the spectrum was made so impure as to blot them out entirely. A physiological phenomenon, however, caused the greatest difficulty. It is evident that, if the adjustments are so made that the two spectra are tangent, the slightest change in focus will destroy the adjustment. Now, the eye seems to dislike the condition of tangency, and, as the two spectra came near together, the eye would suddenly change its focus and they would overlap. I believe this could be much improved by using a collimator of very small angular aperture.

TABLE I

| | $\lambda=486$ | $\lambda=434$ | $\lambda=410$ |
|---|---|---|---|
| Old glass | 0.46 | 0.23 | 0.07 |
| 3670 | 0.89 | 0.65 | 0.52 |
| 3892 | 0.82 | 0.72 | 0.58 |
| 3631 | 0.89 | 0.73 | 0.61 |

The difference between the coefficients of absorption of the new glasses was too small to be of any moment, so I selected No. 3670, since this glass gave the greatest dispersion of the three. The indices of refraction of this glass, interpolated from values furnished me by Mr. Brashear, are 1.6287 for $\lambda=486$, 1.6389 for $\lambda=434$, and 1.6453 for $\lambda=410$. For the old prism $n=1.728$ for $\lambda=434$. With these data we find the most suitable prism-angle to be $62^\circ$ 48′. The effective aperture of the collimator was about 25 mm, and as I expected to work between the limits $\lambda=4600$ and $\lambda=4100$, I determined the sizes of the prisms so that they would transmit a clear beam of 30 mm diameter between the above wave-length limits. This gave for the sizes of the faces of the prisms:

No. 1 = 30 mm × 58 mm
No. 2 = 30 mm × 61 mm
No. 3 = 30 mm × 66 mm

leaving but small leeway from theory. The prisms were ordered from Mr. Brashear and were found most satisfactory. Computing all losses, we find that the intensity of light ($H\gamma$) transmitted

through the two dense-flint prisms is 0.263, and through the three light ones, 0.338. The dispersion along the plate was, for the old, 0.048 mm per Ångström unit, and for the new, 0.054 mm. From this it is evident that the light efficiency of the new in terms of the old is $\frac{0.338\times 0.048}{0.263\times 0.054}=1.15$, or a gain of 15 per cent. This, combined with the gain of 12 per cent. in dispersion, seemed to indicate a very decided improvement, especially when it is evident that the gain on the violet side of $H\gamma$ would be much more marked.

By this time the money originally appropriated by the board of trustees for the new prisms was gone, except just enough to purchase the optical parts, and I was compelled to build the mounting myself. The prisms were fastened in a fixed position at minimum deviation for $H\gamma$. The Potsdam method of using the light reflected from the front face of the first prism was used for following. The following telescope was of ample aperture to transmit the full beam, and was provided with a single spider-web at right angles to the image of the slit, thus allowing the entire length of the slit to be uncovered during the exposure on the star whose image was kept bisected by the spider-web. The cross-wire was illuminated by a small incandescent lamp, controlled by a rheostat in easy reach of the observer. Before reaching the cross-wire the light passed through two thicknesses of blue glass, and a third piece was placed between the eyepiece and the eye. The new and the old instruments were carefully tested, and, so far as I could tell, confirmed the above figures for $H\gamma$, though 15 per cent. is not easy to detect in this way. Suffice it to say that satisfactory spectrograms have been secured in 60 minutes of $\eta$ *Piscium*, whose photographic magnitude is 5.02.

### COMPARISON APPARATUS

The comparison apparatus carried a rocking arm to one side of which was fastened the spark terminals, and to the other the Plücker tube. A spring catch enabled either tube or spark to be brought into the axis of collimation of the telescope without disturbing the position of the condensing lens which concentrated the light on the slit. The capillary of the Plücker tube and the axis of the spark

were both parallel to the slit. Between tube or spark and condensing lens was placed a piece of ground glass—a suggestion of the late Professor Keeler's.

During the exposure on the star the entire apparatus could be turned on a hinge so as to be out of the way. When the work was first started, the iron and hydrogen were mounted on separate supports; no condensing lens was used for the hydrogen, whose capillary was placed at right angles to the slit. These plates showed a small but persistent difference of the artificial $H\gamma$ relative to the iron lines, which practically disappeared with the newer apparatus. The end of the collimator carried an arm which could be turned down on the slit so as to cover up that portion occupied by the star, leaving two narrow openings on each side. This occulting bar was rather wide, so that the plates showed quite a gap of clear glass between the star spectrum and the comparison spectrum. The camera carried an occulting bar, as described by Campbell, the bar covering up the more intense comparison lines during part of the exposure. The comparison spectrum was photographed both at the beginning and the end of the exposure on the star, except in a few cases, where this latter was under two minutes. During an evening's work the spectroscope was inclosed in a box of $\frac{1}{4}$-inch pine, heavily oiled and varnished, on the inside of which were coils for electric heating. These coils were used in only a few of the earlier plates. Temperatures were read at the beginning and end, or if the plates followed rapidly, only at the beginning. The record of the observations is given in Table II. The temperatures, where incomplete, were filled out from the plates either immediately preceding or following the one listed.

## TABLE II

RECORD OF PHOTOGRAPHS

| Plate No. | Date of Photograph | Name of Star | Focus | Prism Temp. Begin | Prism Temp. End | Box Temp. Begin | Box Temp. End | Sky | Exposure | Sid. Time of End | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 534 | Nov. 2, 1898 | *α Cassiopeiae* | 25.9 | .. | .. | .. | .. | Hazy | $45^m$ | $2^h$ $40^m$ | Temperature of air 37°.5 Temperature of prisms 43°. Comparison *Hγ*. Potsdam reduction, using two negatives together. |
| 535 | " 7, " | " | 26.2 | .. | .. | .. | .. | Clear | 35 | 1 45 | Prism temperature 46°. Comparison as in 534 |
| 536 | " 11, " | " | 25.7 | .. | .. | .. | .. | " | 40 | 1 35 | Temperature of air 33°. Prism temperature 37°. Comparison as above. |
| 539 | " 20, " | " | 26.1 | .. | .. | .. | .. | " | 40 | 1 40 | Temperature of air 42°.5. Prism temperature 46°. Comparison as above. |
| 540 | Dec. 8, " | " | 24.9 | .. | .. | .. | .. | " | 40 | 2 05 | Air temperature 12°.5. Prism temperature 18.° Comparison as above. |
| 562 | Jan. 22, 1899 | *α Aurigae* | 25.7 | .. | .. | .. | .. | L't cl'ds V. thick | 60 | 6 25 | Lantern slide plate. Comparison *Fe* and *H*. Air T=35°. |
| 563 | " 25, " | " | 25.7 | .. | .. | .. | .. | " | 60 | 6 35 | Lantern slide plate. Comparison as above. Air temperature, 33°. |
| 564 | " 29, " | " | 24.9 | .. | .. | .. | .. | Clear | 60 | 6 55 | *H* and *Fe* comparison. Lantern slide plate. Air temperature 13° to 10°. |
| 572 | May 13, " | *η Boötis* | 26.7 | 59°.0 | 58°.3 | .. | .. | " | 45 | 13 05 | *Fe* and *H* |
| 581 | " 24, " | " | 27.0 | 68.0 | 66.0 | .. | .. | V. Hazy | 60 | 13 50 | " |
| 585 | June 2, " | " | 27.4 | 73.0 | 71.0 | .. | .. | Clear | 40 | 15 30 | " |
| 586 | " 10, " | " | 27.0 | .. | .. | .. | .. | " | 60 | 15 10 | " Air temperature, 65°. |
| 587 | " 17, " | " | 27.1 | .. | .. | .. | .. | " | 60 | 15 35 | *Fe* and *H*. Air temperature, 68°. |
| 588 | " 18, " | " | 27.1 | .. | .. | .. | .. | " | 35 | 15 05 | *Fe* and *H*. Air temperature, 70°. |
| 591 | " 29, " | " | 27.0 | .. | .. | .. | .. | " | 45 | 16 15 | " |
| 595 | Jan. 21, 1900 | *κ Geminorum* | 25.7 | 30 | 28 | .. | .. | " | 65 | 9 10 | " |
| 601 | " 26, " | " | 25.3 | 19.5 | 18.0 | .. | .. | " | 60 | 7 45 | " |
| 602 | " 28, " | " | 25.1 | 16 | 12 | .. | .. | | 60 | 7 32 | " |
| 603 | Feb. 13, " | " | 25.6 | 30 | 29.5 | .. | .. | " | 60 | 7 40 | " |
| 604 | " 13, " | *ο Ursae Majoris* | 25.6 | 29.5 | 28.5 | .. | .. | " | 60 | 9 40 | " Took out following eyepiece during exposure; moved hard. |
| 605 | " 19, " | *κ Geminorum* | 25.3 | 23 | 21 | .. | .. | " | 60 | 7 50 | |
| 606 | " 19, " | *ο Ursae Majoris* | 25.3 | 21 | 21 | .. | .. | " | 40 | 9 20 | *Fe* and *H*; and seeing very bad on 605 and 606 |
| 611 | Mar. 21, " | " | 25.7 | 31 | 31 | .. | .. | " | 50 | 9 30 | *Fe* and *H*. |
| 613 | " 22, " | " | 26.4 | 47.5 | 46.0 | .. | .. | " | 50 | 8 25 | " Seeing v. good. |
| 615 | " 31, " | " | 26.4 | 42 | 40 | .. | .. | " | 50 | 9 35 | " |
| 617 | Apr. 4, " | " | 26.4 | 41 | 39 | .. | .. | " | 50 | 9 30 | " |
| | | | | | | | | | | | All above plates taken with battery of two dense flint prisms, covered with bag of velvet and felt. All following plates taken with battery of three prisms, and measured and reduced asdescribed |
| 681 | Apr. 20, 1902 | *α Ursae Majoris* | 24.9 | 62 | 61.5 | 61 | 60.5 | V. hazy | 45 | 12 45 | |
| 682 | " 22, " | " | 24.3 | — | 77 | 77 | 76 | Hazy | 40 | 11 05 | Heating coils used. |
| 683 | " 23, " | " | 25.1 | 55 | 55 | 54 | 55 | Clear | 45 | 11 10 | Light clouds passing. |
| 685 | " 23, " | *η Boötis* | 25.1 | 53 | 53 | 52 | 51 | " | 30 | 14 40 | |
| 686 | " 23, " | *α Boötis* | 25.1 | — | — | — | — | " | 4 | 14 55 | |
| 687 | " 24, " | *α Ursae Majoris* | 25.1 | 62.5 | 62 | 61 | 61 | V. Clear | 30 | 11 00 | |
| 690 | " 27, " | " | 25.1 | 62 | 63 | 60 | 60 | Clear | 30 | 11 10 | Seeing fine. |

TABLE II—*Continued*

| Plate No. | Date of Photograph | Name of Star | Focus | Prism Temp. | | Box Temp. | | Sky | Exposure | Sid. Time of End | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Begin | End | Begin | End | | | | |
| 692 | Apr. 27, 1902 | *η Boötis* | 25.1 | 57° | 56° | 55° | 54° | Clear | $25^m$ | $14^h\ 35^m$ | |
| 696 | “ 30, “ | *β Virginis* | 24.9 | 62 | 62 | 61 | 61 | “ | 70 | 13 30 | Induction coil broke. |
| 699 | May 2, “ | *η Boötis* | 24.3 | 74 | 73.5 | 73 | 74 | “ | 25 | 14 40 | Heating coils used. |
| 700 | “ 2, “ | *α Boötis* | 24.3 | — | — | — | — | “ | 3 | 15 08 | |
| 706 | “ 8, “ | *β Virginis* | 24.7 | 69.5 | 69.5 | 68.5 | 68 | “ | 75 | 13 45 | One heating coil used. |
| 707 | “ 8, “ | *η Boötis* | 24.7 | 69 | 69 | 70 | 68 | “ | 30 | 14 05 | One heating coil used; second coil 5 minutes. |
| 709 | “ 9 “ | *β Virginis* | 25.1 | 53.5 | 54 | 52.5 | 53 | “ | 75 | 13 20 | One heating coil. |
| 710 | “ 9, “ | *η Boötis* | 25.1 | 53.5 | 53.5 | 52 | 52.5 | “ | 30 | 14 35 | One and two coils. |
| 711 | “ 9, ” | *α Boötis* | 25.1 | — | — | — | — | “ | 4 | 14 53 | One coil. |
| 713 | “ 12, “ | *ε Virginis* | 24.3 | 74 | 72 | 72 | 70 | “ | 75 | 14 40 | One and two coils. |
| 714 | “ 12, “ | *α Boötis* | 24.3 | — | — | — | — | “ | 3 | 14 55 | One coil. |
| 716 | “ 26, “ | *β Virginis* | 24.7 | 67 | 66.5 | 66 | 65 | “ | 70 | 13 30 | One coil part time. |
| 717 | “ 28, “ | “ | 24.7 | 58 | 57.5 | 68.5 | 68 | “ | 70 | 13 40 | One coil part time. |
| 719 | June 9, “ | *ε Virginis* | 24.7 | 70.5 | 68 | 69.5 | 69 | “ | 60 | 14 40 | |
| 720 | “ 9, “ | *α Boötis* | 24.7 | 68 | — | 67 | — | “ | 4 | 15 24 | One coil. |
| 722 | “ 11, “ | *ε Virginis* | 24.1 | 81 | 80.5 | 80 | 80 | “ | 60 | 14 35 | One coil part time. |
| 723 | “ 16, “ | “ | 24.5 | 72.5 | 72.5 | 72 | 71 | “ | 60 | 14 45 | One coil part time |
| 725 | “ 21, “ | “ | 24.7 | 65.5 | — | 64.5 | — | “ | 60 | 15 10 | One coil part time. |
| 726 | “ 21, “ | *α Boötis* | 24.7 | — | — | — | — | “ | 4 | 15 30 | |
| 754 | July 22, 1903 | *β Draconis* | 24.5 | — | 72 | 72 | 71 | V. clear | 50 | 18 30 | |
| 756 | “ 23, “ | *δ Draconis* | 24.5 | 74 | 74 | 75 | 73 | “ | 75 | 19 20 | |
| 758 | “ 24, “ | *β Draconis* | 24.3 | 81 | 80 | 81 | 79 | Hazy | 75 | 17 45 | |
| 761 | “ 31, “ | “ | 24.7 | 68 | 67 | 68 | 66 | Clear | 60 | 19 00 | |
| 764 | Aug. 7, “ | “ | 24.7 | 64 | — | 64 | — | “ | 60 | 19 20 | Seeing very bad. |
| 765 | “ 7, “ | *δ Draconis* | 24.7 | — | 62 | — | 61 | “ | 75 | 20 55 | Clouds at end. |
| 766 | “ 9, “ | *β Draconis* | 24.7 | — | 73 | 74 | 72 | “ | 45 | 18 35 | Light clouds. |
| 768 | “ 11, “ | *δ Draconis* | 24.5 | 68 | — | 70 | — | “ | 60 | 19 15 | Through clouds. |
| 770 | “ 12, “ | *ε Cygni* | 24.5 | 65 | 64 | 64 | 64 | L't cl'ds | 35 | 20 40 | |
| 771 | “ 16, “ | *δ Draconis* | 24.5 | 74 | 74 | 74 | 72 | Clear | 75 | 19 20 | |
| 772 | “ 16, “ | *ε Cygni* | 24.5 | 72 | 71 | 71 | 70 | “ | 35 | 20 45 | |
| 774 | “ 17, “ | *δ Draconis* | 24.5 | 76 | 74 | 76 | 74 | “ | 70 | 19 20 | |
| 775 | “ 17, “ | *ε Cygni* | 24.5 | 74 | 73 | 73 | 72 | “ | 40 | 20 45 | |
| 776 | “ 17, “ | *η Pegasi* | 24.5 | 72 | 70 | 71 | 69 | Hazy | 60 | 22 35 | |
| 780 | “ 20 “ | *ε Cygni* | 24.5 | — | — | — | — | Clear | 40 | 20 50 | |
| 781 | “ 20, “ | *η Pegasi* | 24.5 | 68 | 66 | 67 | 65 | “ | 60 | 22 30 | |
| 782 | “ 21, “ | *γ Cygni* | 24.5 | 76 | 76 | 76 | 76 | “ | 20 | 19 20 | |
| 783 | “ 21, “ | *ε Cygni* | 24.5 | 75 | 74 | 75 | 73 | “ | 45 | 20 45 | |
| 784 | “ 21, “ | *η Pegasi* | 24.5 | 74 | 73 | 73 | 72 | “ | 60 | 22 30 | |
| 785 | “ 23, “ | *γ Cygni* | 23.9 | 86 | — | 86 | — | “ | 20 | 20 10 | |
| 787 | Sept. 1, “ | *χ Draconis* | 24.7 | 68 | 68 | 68 | 67 | Hazy | 60 | 20 05 | |
| 788 | “ 1, “ | *γ Cygni* | 24.7 | 68 | 66 | 67 | 65 | Clear | 25 | 21 00 | |
| 789 | “ 1, “ | *η Pegasi* | 24.7 | 66 | 64 | 64 | 62 | Hazy | 60 | 22 35 | |
| 790 | “ 2, “ | *χ Draconis* | 24.5 | 73 | 73 | 73 | 72 | V. hazy | 60 | 19 45 | |
| 792 | “ 6, “ | *γ Cygni* | 24.7 | 65 | 66 | 66 | — | Clear | 25 | 19 55 | |
| 793 | “ 6, “ | *η Pegasi* | 24.7 | 66 | 64 | 64 | 63 | “ | 60 | 21 53 | Clouds at end. |
| 794 | “ 12, “ | *γ Cygni* | 24.1 | 80 | — | 80 | — | “ | 30 | 20 25 | |
| 799 | “ 24, “ | *ε Pegasi* | 25.3 | 54 | 53 | 54 | 52 | “ | 60 | 21 45 | |
| 800 | “ 24, “ | *δ Cephei* | 25.3 | 53 | 51 | 52 | 50 | “ | 75 | 00 00 | |
| 802 | “ 25, “ | *ε Pegasi* | 24.9 | 63 | 62 | 62 | 61 | V. sm'ky | 75 | 22 00 | |
| 803 | “ 25, “ | *δ Cephei* | 24.9 | 62 | 59 | 61 | 58 | “ | 75 | 00 00 | |
| 807 | Oct. 12, “ | *α Arietis* | — | 53 | 50 | 52 | 50 | Clear | 60 | 2 15 | Windows badly dewed; objective O. K. |
| 808 | “ 18, “ | *ε Pegasi* | 25.3 | 49 | — | 59 | — | “ | 75 | 23 05 | |
| 811 | “ 19, “ | “ | 25.1 | 58 | 58 | 58 | 57 | Hazy | 75 | 23 15 | |
| 812 | “ 24, “ | “ | 25.7 | 41 | 40 | 40 | 40 | Clear | 75 | 23 30 | |
| 814 | “ 24, “ | *α Arietis* | 25.7 | — | 39 | — | 38 | “ | 60 | 2 05 | Charged batteries during exposure. |
| 816 | “ 25, “ | “ | 25.5 | 48 | 48 | 48 | 48 | “ | 60 | 1 45 | |
| 819 | “ 26, “ | “ | 25.5 | 35 | 34 | 34 | 34 | “ | 60 | 3 10 | |
| 820 | “ 28, “ | “ | 25.8 | 44 | 44 | 44 | 44 | “ | 60 | 1 25 | |
| 848 | Mar. 23, 1904 | *β Geminorum* | 23.3 | 40 | 40 | 40 | 40 | “ | 30 | 8 30 | |
| 852 | “ 28, “ | “ | 23.9 | 36 | 36 | 36 | 36 | V. hazy | 15 | 9 05 | Seeing very bad. |
| 857 | Apr. 4, “ | “ | 23.7 | 41 | 41 | 41 | 41 | “ | 30 | 9 45 | April 18 found camera focus 1 mm out. |
| 858 | “ 5, “ | “ | 23.3 | 53 | 53 | 53 | 53 | “ | 30 | 9 20 | |
| 860 | “ 14, “ | “ | 23.7 | 42 | — | 42 | 42 | Clear | 25 | 10 05 | |

TABLE II—*Continued*

| Plate No. | Date of Photograph | Name of Star | Focus | Prism Temp. Begin | Prism Temp. End | Box Temp. Begin | Box Temp. End | Sky | Exposure | Sid. Time of End | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 861 | Apr. 14, 1904 | γ *Leonis* | 23.7 | 41 | 41 | 41 | 40 | Clear | $50^m$ | $11^h$ $35^m$ | |
| 862 | " 16, " | " | 23.7 | 37 | 38 | 37 | 37 | V. Clear | 45 | 11 05 | |
| 864 | " 19, " | " | 24.0 | 33 | 34 | 33 | 33 | Clear | 60 | 11 20 | |
| 668 | May 5, " | " | 22.5 | 72 | 70 | 72 | 70 | —— | 60 | 11 45 | |
| 870 | " 9, " | " | 23.0 | 60 | 59 | 59 | 59 | Clear | 60 | 12 15 | |
| 871 | " 10, " | α *Boötis* | 23.3 | 50 | 50 | 50 | 50 | " | 4 | 13 16 | |
| 872 | " 10, " | ε *Boötis* | 23.3 | 50 | 49 | 50 | 49 | —— | 45 | 14 45 | Through clouds. |
| 876 | " 20, " | " | 25.0 | 58 | 58 | 58 | 58 | —— | 30 | 15 50 | |
| 877 | " 27, " | " | 23.7 | 65 | 65 | 65 | 64 | Clear | 45 | 14 25 | |
| 878 | " 27, " | α *Serpentis* | 23.7 | 65 | 63 | 64 | 61 | " | 60 | 15 40 | |
| 879 | " 27, " | ζ *Herculis* | 23.7 | 63 | 60 | 61 | 59 | " | 45 | 16 45 | |
| 881 | " 28, " | α *Boötis* | 23.7 | 66 | — | 66 | — | " | 4 | 13 15 | |
| 882 | " 28, " | ε *Boötis* | 23.7 | 66 | 66 | 66 | 65 | " | 45 | 14 25 | |
| 884 | " 28, " | ζ *Herculis* | 23.7 | 64 | 62 | 62 | 61 | " | 45 | 16 50 | |
| 885 | June 3, " | ε *Boötis* | 23.1 | 81 | — | 81 | — | Clouds | 45 | 14 30 | |
| 887 | " 9. " | ζ *Herculis* | 23.9 | 64 | 63 | 63 | 62 | V. hazy | 45 | 16 55 | Seeing bad. |
| 889 | " 11, " | α *Serpentis* | 23.5 | 69 | 69 | 70 | 68 | —— | 60 | 15 55 | |
| 892 | " 12, " | " | 23.3 | 73 | 72 | 72 | 71 | Clear | 60 | 15 45 | |
| 895 | " 16, " | " | 23.7 | — | 66 | — | 66 | " | 70 | 15 50 | |
| 896 | " 16, " | ζ *Herculis* | 23.7 | 66 | 65 | 66 | 64 | —— | 45 | 16 55 | Heavy fog low down. |
| 897 | " 17, " | α *Serpentis* | 23.5 | 70 | 70 | 70 | 69 | Clear | 60 | 15 50 | |
| 898 | " 17, " | ζ *Herculis* | 23.5 | 70 | 68 | 69 | 67 | " | 45 | 17 00 | |
| 903 | Sept. 10, " | β *Cygni* | 24.3 | 73 | 72 | 73 | 72 | —— | 75 | 21 05 | |
| 904 | " 13, " | " | 24.3 | — | 70 | — | 70 | V. hazy | 75 | 20 40 | |
| 905 | " 14, " | " | 24.9 | 58 | 58 | 58 | 58 | Clear | 75 | 21 20 | |
| 906 | " 15, " | " | 24.9 | 60 | 60 | 59 | 58 | —— | 75 | 20 45 | Clear at beginning, hazy at end. |
| 907 | " 17, " | " | 24.3 | 74 | — | 74 | 73 | V. hazy | 75 | 21 15 | |
| 909 | Oct. 15, " | δ *Andromedae* | 25.3 | 48 | 48 | 48 | 48 | Clear | 60 | 0 45 | |
| 910 | " 15, " | γ *Anaromedae* | 25.3 | — | — | — | — | " | 45 | 2 00 | |
| 013 | " 17, " | δ *Andromedae* | 25.0 | 60 | 58 | 59 | 57 | V. smoky | 60 | 0 30 | |
| 014 | " 18, " | " | 24.0 | 59 | 58 | 59 | 58 | " | 60 | 0 30 | |
| 016 | " 21, " | " | 24.5 | 45 | 46 | 45 | 46 | Clear | 60 | 0 55 | |
| 017 | " 24, " | " | 24.3 | 52 | 52 | 52 | 52 | Hazy | 60 | 0 10 | |
| 020 | " 29, " | γ *Andromedae* | 24.3 | 48 | 47 | 47 | 46 | —— | 45 | 1 35 | Clear but smoky. |
| 022 | " 29, " | α *Persei* | 24.3 | 45 | 45 | 44 | 44 | Clear | 10 | 3 00 | |
| 025 | " 30, " | η *Cassiopeiae* | 24.5 | 45 | 44 | 45 | 44 | V. clear | 60 | 0 20 | |
| 026 | " 30, " | γ *Andromedae* | 24.5 | 44 | 43 | 44 | 42 | " | 45 | 1 20 | |
| 027 | " 30, " | γ *Persei* | 24.5 | 43 | 42 | 42 | 41 | " | 45 | 2 20 | |
| 028 | " 30, " | α *Persei* | 24.5 | 42 | 41 | 41 | 40 | " | 10 | 2 44 | |
| 930 | " 31, " | η *Cassiopeiae* | 24.5 | 45 | 45 | 45 | 45 | Smoky | 60 | 0 15 | |
| 931 | " 31, " | γ *Andromedae* | 24.5 | 45 | 44 | 45 | 43 | V. smoky | 45 | 1 15 | |
| 932 | " 31, " | γ *Persei* | 24.5 | 44 | 42 | 43 | 42 | " | 45 | 2 20 | |
| 936 | Nov. 6, " | η *Cassiopeiae* | 24.7 | 40 | 40 | 40 | 40 | Clear | 60 | 0 20 | |
| 938 | " 6, " | γ *Persei* | 24.7 | 39 | 40 | 39 | 38 | V. clear | 45 | 2 15 | |
| 939 | " 6, " | γ *Andromedae* | 24.7 | 40 | 39 | 38 | 38 | " | 45 | 3 15 | |
| 940 | " 6, " | α *Persei* | 24.7 | — | — | — | — | —— | 10 | 3 35 | |
| 941 | " 11, " | η *Cassiopeiae* | 25.1 | 36 | 36 | 36 | 36 | V. clear | 60 | 0 50 | |
| 942 | " 11, " | γ *Persei* | 25.1 | 36 | 36 | 36 | 35 | " | 45 | 1 50 | |
| 943 | " 11, " | α *Persei* | 25.1 | 36 | 36 | 35 | 35 | " | 10 | 2 18 | |
| 945 | " 22, " | η *Cassiopeiae* | 24.5 | 45 | 46 | 45 | 44 | Clear | 60 | 0 55 | |
| 946 | " 22, " | γ *Persei* | 24.5 | — | — | — | — | " | 45 | 2 40 | Light clouds at end. |
| 947 | " 22, " | α *Persei* | 24.5 | — | 42 | — | 40 | " | 10 | 3 10 | |
| 948 | " 30, " | η *Piscium* | 25.1 | 30 | 30 | 30 | 28 | " | 75 | 2 00 | |
| 949 | " 30, " | α *Tauri* | 25.1 | 29 | 28 | 28 | 27 | " | 30 | 3 25 | |
| 951 | Dec. 14, " | " | 25.5 | 18 | 18 | — | — | " | 30 | 3 35 | |
| 953 | " 30, " | η *Piscium* | 24.7 | 39 | 37 | 38 | 37 | " | 75 | 2 55 | |
| 954 | " 30, " | α *Tauri* | 24.7 | 37 | 38 | 37 | 37 | " | 30 | 3 40 | |
| 956 | " 31, " | η *Piscium* | 24.7 | 45 | 44 | 45 | 43 | " | 75 | 2 55 | |
| 957 | " 31, " | α *Tauri* | 24.7 | 44 | 44 | 43 | 43 | " | 30 | 3 40 | |
| 960 | Jan. 1, 1905 | η *Piscium* | 24.3 | 50 | 49 | 50 | 48 | " | 75 | 2 40 | |
| 961 | " 1, " | α *Tauri* | 24.3 | 49 | 49 | 48 | 48 | " | 30 | 3 25 | |

TABLE II—*Continued*

PLATES OF *Venus*

| Plate No. | Date of Photograph | Focus | Prism Temp. | | Box Temp. | | Sky | Exposure | Sid. Time of End | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Begin | End | Begin | End | | | | |
| 651 | Nov. 27, 1901 | 25.9 | 35 | — | — | — | Clear | $30^s$ | $21^h$ $40^m$ | |
| 658 | " 30, " | 25.5 | 46 | — | — | — | V. smoky | 60 | | Sid. T. end about $22^h$ 00. |
| 660 | Dec. 4, " | 26.0 | 33 | — | — | — | Clear | 40 | 22 23 | |
| 661 | " 15, " | 26.7 | 15 | — | — | — | " | 45 | 23 00 | |
| 666 | " 30, " | 26.0 | 38 | — | — | — | " | 45 | 0 15 | |
| 668 | " 31, " | 26.0 | 42 | — | — | — | " | 45 | 23 50 | Position circle, 91°. |
| 673 | Jan. 4, 1902 | 26.0 | 28 | — | — | — | " | 45 | 0 25 | |
| 676 | " 6, " | 26.0 | 39 | — | — | — | Smoky | 45 | 0 30 | |
| 689 | Apr. 26, " | 25.1 | 46 | — | — | — | Clear | 30 | | 4:30 A. M. Mean Time. |
| 695 | " 29, " | 25.1 | — | — | — | — | " | 6 | 20 20 | |
| 705 | May 5, " | 24.3 | — | — | — | — | " | 60 | 19 45 | |
| 731 | Apr. 10, 1903 | 24.7 | 65 | — | — | — | " | 90 | 8 40 | Position circle, 92°. |
| 734 | " 18, " | 25.0 | 58 | — | — | — | " | 180 | | Position circle, 92°, 7:30 P. M., Mean Time. |
| 747 | July 5, " | 23.8 | 84 | — | — | — | —— | 120 | | Position circle, 91°, 8:00 P. M. |
| 748 | " 6, " | 23.8 | 87 | — | — | — | —— | 45 | 15 25 | |
| 749 | " 7, " | 23.8 | 84 | — | — | — | —— | 60 | 15 00 | |
| 751 | " 8, " | 23.8 | 84 | — | — | — | —— | 90 | 15 00 | |
| 952 | Dec. 30, 1904 | 24.7 | — | — | 40 | — | —— | 45 | 0 25 | |
| 955 | " 31, " | 24.7 | — | — | 46 | — | —— | 40 | .— | 5:30 P. M., Mean Time. |
| 962 | Jan. 16, 1905 | 25.3 | 27 | — | — | — | Clear | 30 | .— | 5:20 " " " |
| 964 | " 26, " | 25.5 | 20 | — | — | — | —— | 35 | .— | 5:40 " " " |
| 965 | Feb. 4, " | 25.5 | — | — | — | — | —— | — | .— | 6:55 " " " |
| 966 | " 7, " | 25.5 | 20 | — | — | — | —— | 30 | .— | 5:30 " " " |
| 967 | " 10, " | 25.5 | 20 | — | — | — | —— | 30 | .— | 5:45 " " " |
| 968 | " 15, " | 27.0 | 8 | — | — | — | —— | 30 | .— | 6:10 " " " |

## ADOPTED WAVE-LENGTHS OF COMPARISON LINES

The wave-lengths of the comparison lines as finally adopted are given below in Table III:

TABLE III

| λ | m | λ | m | λ | m |
|---|---|---|---|---|---|
| 4181.92..... | 40.654 | 4227.61.... | 43.557 | 4340.63.... | 50.001 |
| 4187.22..... | 41.000 | 4260.66.... | 45.544 | 4383.72.... | 52.217 |
| 4187.97..... | 41.049 | 4271.93.... | 46.202 | 4404.93.... | 53.264 |
| 4202.20..... | 41.966 | 4308.07.... | 48.244 | 4415.30.... | 53.766 |
| 4219.52..... | 43.057 | 4325.94.... | 49.218 | | |

The wave-lengths of the iron lines were at first taken from Rowland's tables, but later on they were corrected by the values published by Kayser.[1] The lines at $\lambda = 4187.971$ and $\lambda = 4308.073$ could never be made to fit with the other iron lines on the star plates, and as this observatory has a five-foot concave grating spectroscope, and as their wave-lengths were not given by Kayser, I determined

[1] *Astrophysical Journal*, **13**, 332, 1901.

to measure their wave-lengths, interpolating from those given by Kayser. Five plates were taken, and the entire list of thirteen iron lines was measured. The eleven lines given by Kayser were regarded as known, and an interpolating curve of the second degree passed through them, the constants of this curve being determined by the method of least squares. An unknown correction, $\Delta\lambda$, was then given to the approximate values $\lambda$ 4187.94 and $\lambda$ 4308.08, and its value computed for these lines, as were also the wave-lengths of the known lines. A different curve was computed for each plate. The values of $\Delta\lambda$ and the residuals of the known lines are given below in Table IV:

TABLE IV

| $\lambda$ | $\Delta\lambda$ | | | | | |
|---|---|---|---|---|---|---|
| | Plate No. 1 | Plate No. 2 | Plate No. 3 | Plate No. 4 | Plate No. 5 | Mean |
| 4181.918 .......... | +0.001 | −0.008 | +0.006 | −0.005 | −0.014 | −0.004 |
| 4187.221 .......... | −0.003 | −0.010 | −0.006 | −0.005 | −0.004 | −0.006 |
| 4187.94 ........... | +0.022 | +0.032 | +0.030 | +0.040 | +0.029 | +0.031 |
| 4202.195 .......... | −0.009 | −0.005 | −0.002 | ±0.000 | −0.005 | −0.004 |
| 4219.523 .......... | +0.009 | −0.017 | +0.024 | +0.006 | +0.021 | +0.009 |
| 4227.606 .......... | +0.008 | +0.021 | −0.024 | +0.015 | +0.018 | +0.008 |
| 4260.656 .......... | −0.002 | +0.001 | −0.001 | −0.005 | −0.001 | −0.002 |
| 4271.933 .......... | + 0.003 | −0.001 | +0.005 | −0.005 | ±0.000 | ±0.000 |
| 4308.08 ........... | −0.009 | −0.002 | −0.011 | −0.002 | −0.013 | −0.007 |
| 4325.941 .......... | −0.013 | −0.002 | +0.001 | −0.004 | −0.001 | −0.004 |
| 4383.724 .......... | +0.003 | −0.018 | −0.007 | +0.002 | −0.021 | −0.008 |
| 4404.929 .......... | +0.006 | +0.005 | +0.017 | +0.002 | +0.016 | +0.009 |
| 4415.301 .......... | −0.005 | +0.013 | −0.011 | −0.002 | +0.004 | ±0.000 |

Applying these corrections, we find the values $\lambda$4187.971 and $\lambda$4308.073, as against $\lambda$4187.943 and 4308.081 as given by Rowland. The values given in Table III were used after April 1, 1903; the earlier plates were not corrected, as the change was too small to be of any moment.

## METHOD OF MEASURING THE PLATES

The plates were all measured upon a Zeiss comparator reading directly to 0.001 mm and by estimation to 0.0001 mm. The division errors were determined over the portion of the scale used, by a method slightly modified from that published by Dr. Gill in *Monthly Notices*, **49**, 105.

Each plate was first carefully examined and, if found satisfactory, was measured and reduced; if unsatisfactory, it was rejected before measuring. In only one case was a plate rejected after measuring. In this case an error in the exposures on the iron comparison was suspected before measuring. This plate was reduced as far as the comparison lines only, $H\gamma$ did not agree with the iron lines, and the plate was then rejected. The plates were placed on the table of the measuring engine so that $H\gamma$ fell very near to scale reading 50, which could be easily done within a few thousandths of a millimeter. Four to six pointings were then made upon the artificial $H\gamma$, and the plate was run to one end, and star lines and comparison lines were measured as they came into the field of view, three pointings on each star line and four pointings on each iron line. As soon as $H\gamma$ was reached, four or six more pointings were made upon it, and the measuring was continued until the end of the plate was reached. The instrument was then set back to $H\gamma$, and four or six more pointings were made. On all of the artificial lines the pointings were equally divided on both sides of the star spectrum. After a short rest, the observer remeasured the plate with the violet end on the opposite hand; the agreement of the three measures of $H\gamma$ served as a check on the work, and in no case was any progressive shift detected, such as found by Curtis.[1] Division error and error of runs were then applied to 0.0001 mm to the mean of the pointings on each line, after which the last place of decimals was dropped. The difference of the mean of the three settings on $H\gamma$ and 50 was then applied to all lines to reduce to the common zero, $H\gamma = 50.000$.

### METHOD OF REDUCTION

On a lantern-slide plate of the solar spectrum, twenty-six carefully selected lines were identified with lines on Rowland's map. The distances of these lines from $H\gamma$ were measured on three different days in two positions of the plate. Through these points the following curve was passed:

$$m = 131.655\,\mathrm{mm} - \frac{[5.1009\dot{2}92]}{\lambda - 2795.54}\ .$$

Here $m$ is the micrometer setting corresponding to any line of wave-

[1] *Lick Observatory Bulletin* No. 62.

length $\lambda$ when $H\gamma$ reads 50. The residuals were all small, only two of the 26 being 0.004 mm. Later on Mr. Maag computed the curve

$$m = 159.388 - \frac{[3.8956145]}{(\lambda - 3098.00)^{0.6}} ,$$

which reduced $\Sigma vv$ from 66 to 29. This curve has, however, not been used, as it gave when tried on a few plates, practically identical results with the simpler curve. With this simpler curve the values of $m$ for each comparison line were computed and are given in Table III. A list of 120 lines was then measured on the solar tandard, and their character was marked. These lines were measured on two different days in both positions of the instrument and reduced to a common zero, $H\gamma = 50$, after which the average value of $m$ was tabulated. Thus the $m$'s in this table of solar lines were determined by measurement directly, and not by computation from their wave-lengths. This was done in the hope of avoiding the uncertainty in the wave-length to be assigned to a "blend."

If a star plate could be taken under exactly the same conditions as the solar standard, it is evident that the difference of the distance of any line on the star negative from the artificial $H\gamma$, subtracted from the corresponding reading taken from the table of solar lines, would give at once its velocity-displacement. But as this can never be done, it becomes necessary to apply corrections due to these changed conditions. In order to determine these corrections from the measures of the fourteen lines of Table III, let us give to the constants of the Hartmann-Cornu interpolation formula small unknown corrections $\Delta m_o$, $\Delta c$, and $\Delta\lambda_o$. The setting on any artificial line is then given by the equation

$$m = m_o + \Delta m_o + \frac{c + \Delta c}{\lambda - (\lambda_o + \Delta\lambda_o)} .$$

From this it readily follows that the difference of any observed value of $m$ and its value taken from Table II will be given by the equation

$$\text{C.} - \text{O.} = x + (m - 47)y + \frac{(m - 47)^2}{10} z .$$

Here $x$, $y$, and $z$ are unknown, and the value of $m$ used in computing

the coefficients may be taken from Table III. This equation may be more simply written

$$C.-O.=x+by+cz\ .$$

Now, the same artificial lines being measured on all plates, $b$ and $c$ are always the same. Moreover, as soon as $x$, $y$, and $z$ become known, this equation worked backward gives the correction necessary to reduce the star lines to the solar standard. The coefficients $b$ and $c$ have been computed once for all and their values added to the table of 120 solar lines.

To determine the values of $x$, $y$, and $z$, we make use of the comparison lines. Each line gives an equation of the above form, and the fourteen equations may be very easily and rapidly reduced by the method of least squares, as follows: Forming the normals we have—

$$Nx+(\Sigma b)y+(\Sigma c)z=\Sigma\,(C.-O.)\ ,$$
$$(\Sigma b)x+(\Sigma b^2)y+(\Sigma bc)z=\Sigma b(C.-O.)\ ,$$
$$(\Sigma c)z+(\Sigma bc)y+(\Sigma c^2)z=\Sigma c(C.-O.)\ .$$

Solving these, we find—

$$x=\alpha_1\Sigma(C.-O.)+\beta_1\Sigma b(C.-O.)+\gamma_1\Sigma c(C.-O.)\ ,$$
$$y=\alpha_2\Sigma(C.-O.)+\beta_2\Sigma b(C.-O.)+\gamma_2\Sigma c(C.-O.)\ ,$$
$$z=\alpha_3\Sigma(C.-O.)+\beta_3\Sigma b(C.-O.)+\gamma_3\Sigma c(C.-O.)\ .$$

Since the same series of artificial lines are always observed, the Greek letters are always the same and their values may be computed once for all. It is only necessary, therefore, to form the quantities $\Sigma(C.-O.)$, $\Sigma b(C.-O.)$, and $\Sigma c(C.-O.)$ anew for each plate. This can be very easily and rapidly done with Crelle's Table. Then substituting their values, using four-place logarithms, the values of $x$, $y$, and $z$ are rapidly computed. Ten plates give average probable errors of $x$, $y$, and $z$, respectively as $r_x=\pm 0.6$, $r_y=\pm 0.08$, and $r_z=\pm 0.02$. The maximum values of the quantities themselves may be taken as $x=75$, $y=25$, and $z=1.5$, the unit being the thousandth of a millimeter; $z$ might possibly be dropped and the reduction correspondingly simplified. I have, however, preferred to keep it. The values of $x$, $y$, and $z$ were checked by substituting their values in the above equation and computing the value of (C.—O.) for each artificial line. I give below a complete example of this reduction. The heavy-faced figures can be printed once for all, the light-faced

figures must be computed for each plate. Mr. Maag tells me that this computation can be easily made in half an hour. It should be noted that any shrinkage of the film, error of measuring engine, or other source of error is completely eliminated, provided only it can be represented by an equation of the second degree. The lines were all given equal weight, except where it was obviously a case of faulty identification, in which case the doubtful line was rejected. It is hoped at some future date to publish the complete reduction sheets of each plate as a contribution from this observatory, but at present we have no funds for such a purpose. Should that be done, the lines rejected will of course appear. It is impossible, however to give them here.

EXAMPLE

Reduction of Iron Lines. Plate No. 806. Star α *Cassiopeiae*. Violet Left.

| λ | Computed $m$ | Observed $m$ | C.−O. $n$[1] | $m-47$ $b$ | $\frac{(m-47)^2}{10}$ $c$ | $bn$[1] | $cn$[1] | $by$[1] | $cz$[1] | Correction $=x+by+cz$[1] |
|---|---|---|---|---|---|---|---|---|---|---|
| 4181.92... | 40.654 | 40.747 | −93 | −6.35 | +4.03 | +591 | −375 | −64 | +1 | −93 |
| 4187.22... | 41.000 | 41.088 | −88 | −6.00 | +3.60 | +528 | −317 | −60 | +1 | −89 |
| 4187.97... | 41.049 | 41.138 | −89 | −5.95 | +3.54 | +530 | −315 | −60 | +1 | −89 |
| 4202.20... | 41.966 | 42.044 | −78 | −5.03 | +2.53 | +392 | −197 | −50 | +1 | −79 |
| 4219.52... | 43.057 | 43.127 | −70 | −3.94 | +1.55 | +276 | −108 | −40 | 0 | −70 |
| 4227.61... | 43.557 | 43.623 | −66 | −3.44 | +1.18 | +227 | − 78 | −35 | 0 | −65 |
| 4260.66... | 45.544 | 45.590 | −46 | −1.46 | +0.21 | + 67 | − 10 | −15 | 0 | −45 |
| 4271.93... | 46.202 | 46.239 | −37 | −0.80 | +0.06 | + 30 | − 2 | − 8 | 0 | −38 |
| 4308.07... | 48 244 | 48.261 | −17 | +1.24 | +0.15 | − 21 | − 3 | +12 | 0 | −18 |
| 4325.94... | 49.218 | 49.224 | − 6 | +2.22 | +0.49 | − 13 | − 3 | +22 | 0 | − 8 |
| 4340.63... | 50.001 | 50.000 | + 1 | +3.00 | +0.90 | + 3 | + 1 | +30 | 0 | ± 0 |
| 4383.72.. | 52.217 | 52.193 | +24 | +5.22 | +2.72 | +125 | + 65 | +52 | +1 | +23 |
| 4404.93... | 53.264 | 53.231 | +33 | +6.26 | +3.92 | +207 | +129 | +63 | +1 | +34 |
| 4415.30... | 53.766 | 53.727 | +39 | +6.76 | +4.57 | +264 | +178 | +68 | +1 | +39 |
| Sums | | | −493 | | | +3206 | −1035 | | | |

| | | | | | |
|---|---|---|---|---|---|
| Log. $\Sigma n$..... | $2.6928_n$ | Log. $\Sigma n$.... | $2.6928_n$ | Log. $\Sigma n$...... | $2.6928_n$ |
| | **9.3008** | | **7.2859** | | **$8.7819_n$** |
| Sum.. ..... | $1.9936_n$ | Sum........ | $9.9787_n$ | Sum.......... | 1.4747 |
| $\alpha$.......... | −98.6 | $\alpha'$.......... | − 0.95 | $\alpha''$.......... | +29.83 |
| Log. $\Sigma bn$.... | 3.5059 | Log. $\Sigma bn$.... | 3.5059 | Log. $\Sigma bn$...... | 3.5059 |
| | **7.2859** | | **7.5378** | | **5.7046** |
| Sum........ | 0.7918 | Sum........ | 1.0437 | Sum.......... | 9.2105 |
| $\beta$.......... | + 6.2 | $\beta'$.......... | +11.03 | $\beta''$.......... | + 0.16 |
| Log. $\Sigma cn$.... | $3.0148_n$ | Log. $\Sigma cn$.... | $3.0148_n$ | Log. $\Sigma cn$..... | $3.0148_n$ |
| | **8.7819** | | **5.7046** | | **8.4592** |
| Sum........ | 1.7967 | Sum........ | $8.7194_n$ | Sum.......... | $1.4740_n$ |
| $\gamma$.......... | +62.6 | $\gamma'$.......... | − 0.05 | $\gamma''$.......... | −29.79 |
| $\alpha+\beta+\gamma=x$.. | −29.8 | $\alpha'+\beta'+\gamma'=y$ | +10.03 | $\alpha''+\beta''+\gamma''=z$ | + 0.20 |

[1] These numbers are in units of last figure in column of $m$'s, or in thousandths of a millimeter.

## CURVATURE

The curvature of the lines was determined experimentally by placing two photographs of the iron spectrum film to film, the violet end of one opposite the red end of the other, and then shifting the plates until the extremities of the curved lines came in contact. This gave us as the equation of the line $x=0.0085$ mm $y^2$. The value of the constant, computed from the known optical constants of the instruments, came out 0.0080 mm. The agreement was considered satisfactory. By the method of following, the star spectrum was seldom midway between the two halves of the iron spectrum, so that the correction to the final velocity was computed by

$$\Delta v=-[1.0475](z_1^2-z_0^2) \ ,$$

where $z_1$ and $z_0$ are respectively the distances of the comparison spectrum and the star spectrum from the line midway between the two parts of the iron spectrum, and were measured for each plate.

## DISCUSSION OF THE RESULTS

The mean velocity for each star, as given in Table V, except of *η Piscium* and those having fewer than five nights' work, was subtracted from the velocity derived for each plate. This gave 129 residuals, the sum of whose squares came out 1005.59. Treating these as residuals from the mean of a single directly measured quantity, we find the probable error of a single plate to be $0.67\times(1005.59 \div 1.28)^{\frac{1}{2}}=\pm 1.88$ km per sec., or the probable error of the mean of five to be $\pm 0.8$ km per sec. This process is open to criticism and is apt to give too small a value for the probable error. But I believe results derived in this way are much nearer the truth than values of the probable error for each star derived from the residuals upon that star alone. It has been the author's experience that a probable error derived from much less than twenty-five residuals is almost without meaning.

In order to gather an idea of the manner in which this probable error was distributed in the several parts of the work, we may proceed as follows: The velocity is computed for each line by the equation

$$v^{\lambda}=k[m_{\text{sun}}-(m_{\text{star}}+\Delta m)] \ .$$

## TABLE V

### Collected Results

$\alpha = 0^h\ 34^m0$ — *δ Andromedae* — $\delta = +30°\ 19'$

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Red. to ☉ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 909.... | Oct. 15, 1904 | +3.9 | −0.1 | +1.9 | −1.2 | +0.7 | 11 | 12 | Maag |
| 913.... | " 17, " | −1.4 | −2.6 | −2.0 | −2.2 | −4.2 | 13 | 13 | " |
| 914.... | " 18, " | +1.5 | ±0.0 | +0.8 | −2.6 | −1.8 | 17 | 17 | " |
| 916.... | " 21, " | +1.5 | +0.6 | +1.0 | −4.1 | −3.1 | 15 | 15 | " |
| 917.... | " 24, " | +4.4 | +2.4 | +3.4 | −5.4 | −2.0 | 17 | 17 | " |

Mean of Velocities = −2.1

$\alpha = 0^h\ 34^m.8$ — *α Cassiopeiae*[1] — $\delta = +56°\ 0'$

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Red. to ☉ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 534.... | Nov. 2, 1898 | ± 0.0 | − 4.0 | − 2.0 | − 2.0 | − 4.0 | 11 | 12 | Lord |
| 535.... | " 7, " | ± 0.0 | + 2.7 | + 1.4 | − 3.7 | − 2.3 | 11 | 12 | " |
| 536.... | " 11, " | +11.4 | +10.1 | +10.8 | − 5.1 | + 5.7 | 10 | 10 | " |
| 539.... | " 20, " | + 8.7 | + 3.2 | + 6.0 | − 8.1 | − 2.1 | 11 | 10 | " |
| 540.... | Dec. 8, " | +14.0 | +14.4 | +14.2 | −13.6 | + 0.6 | 11 | 11 | " |

Mean of Velocities = −0.4

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Red. to ☉ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 801.... | Sept. 24, 1903 | − 9.9 | −11.5 | −10.7 | +11.4 | + 0.7 | 13 | 13 | Maag |
| 806.... | Oct. 12, " | − 9.9 | − 8.9 | − 9.4 | + 5.9 | − 3.5 | 14 | 14 | " |
| 809.... | " 18, " | − 5.9 | − 7.3 | − 6.6 | + 3.9 | − 2.7 | 15 | 15 | " |
| 813.... | " 24, " | − 3.4 | − 6.0 | − 4.7 | + 1.8 | − 2.9 | 17 | 16 | " |
| 815.... | " 25, " | − 5.9 | − 5.7 | − 5.8 | + 1.4 | − 4.4 | 17 | 17 | " |

Mean of Velocities = −2.6

$\alpha = 0^h\ 43^m.0$ — *η Cassiopeiae* — $\delta = +57°\ 17'$

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Red. to ☉ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 925.... | Oct. 30, 1904 | + 9.4 | + 6.2 | + 7.8 | + 0.4 | + 8.2 | 15 | 15 | Maag |
| 930.... | " 31, " | +10.1 | + 8.8 | + 9.4 | + 0.1 | + 9.3 | 18 | 18 | " |
| 936.... | Nov. 6, " | +12.0 | + 9.9 | +11.0 | − 2.1 | + 8.9 | 19 | 19 | " |
| 941.... | " 11, " | +13.0 | +12.1 | +12.6 | − 3.9 | + 8.7 | 15 | 15 | " |
| 945.... | " 22, " | +16.2 | +16.8 | +16.5 | − 7.7 | + 8.8 | 19 | 19 | " |

Mean of Velocities = +8.8

$\alpha = 1^h\ 26^m.2$ — *η Piscium*[2] — $\delta = +14°\ 50'$

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Red. to ☉ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 948.... | Nov. 30, 1904 | +34.3 | +33.1 | +33.7 | −20.6 | +13.1 | 17 | 17 | Maag |
| 953.... | Dec. 30, " | +47.2 | +43.7 | +45.4 | −28.9 | +16.5 | 17 | 17 | " |
| 956.... | " 31, " | +47.2 | +44.7 | +46.0 | −29.0 | +17.0 | 15 | 15 | " |
| 960.... | Jan. 1, 1905 | +49.5 | +48.0 | +48.8 | −29.1 | +19.7 | 13 | 13 | " |

Mean of Velocities = +16.6

[1] Earlier measures: Vogel, 1890, −14.9; Scheiner, 1890, −15.6; Campbell, 1896, −4.3; Lord, 1897, −0.6. The first series, taken with two 60° dense-flint prisms. Spectroscope covered with felt and velvet bag. Comparison $H\gamma$ only. Measured by placing plate film to film with solar standard.

[2] Suspected variable.

TABLE V—*Continued*

*γ Andromedae*

$\alpha = 1^h\ 57^m.8$ $\delta = +41°\ 51'$

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 910.... | Oct. 15, 1904 | −11.5 | −13.3 | −12.4 | + 8.8 | − 3.6 | 17 | 17 | Maag |
| 920.... | " 29, " | − 9.4 | −12.5 | −11.0 | + 2.7 | − 8.3 | 13 | 13 | " |
| 926.... | " 30, " | − 9.9 | −10.3 | −10.1 | + 2.3 | − 7.8 | 17 | 17 | " |
| 931.... | " 31, " | −10.3 | − 7.0 | − 8.6 | + 1.8 | − 6.8 | 15 | 15 | " |
| 939 ... | Nov. 6, " | − 7.7 | − 8.0 | − 7.8 | − 1.1 | − 8.9 | 17 | 17 | " |

Mean of Velocities = −7.1

*α Arietis*

$\alpha = 2^h\ 01^m.5$ $\delta = +23°\ 0'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 807.... | Oct. 12, 1903 | −20.1 | −20.2 | −20.2 | + 8.5 | −11.7 | 19 | 19 | Maag |
| 814.... | " 24, " | −13.9 | −14.3 | −14.1 | + 2.5 | −11.6 | 18 | 18 | " |
| 816.... | " 25, " | −13.6 | −13.9 | −13.8 | + 2.0 | −11.8 | 17 | 17 | " |
| 819.... | " 26, " | −15.4 | −14.7 | −15.0 | + 1.6 | −13.4 | 19 | 19 | " |
| 820.... | " 28, " | −12.1 | −15.7 | −13.9 | + 0.6 | −13.3 | 16 | 16 | " |

Mean of Velocities = −12.4

*γ Persei*

$\alpha = 2^h\ 57^m.6$ $\delta = +53°\ 7'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 927.... | Oct. 30, 1904 | − 0.1 | − 0.3 | − 0.2 | + 8.8 | + 8.6 | 17 | 17 | Maag |
| 932.... | " 31, " | − 1.9 | − 3.7 | − 2.8 | + 8.4 | + 5.6 | 17 | 17 | " |
| 938.... | Nov 6, " | + 0.7 | − 1.2 | − 0.2 | + 6.0 | + 5.8 | 16 | 15 | " |
| 942.... | " 11, " | − 0.7 | ± 0.0 | − 0.4 | + 4.0 | + 3.6 | 16 | 16 | " |
| 946.... | " 22, " | + 3.4 | + 2.3 | + 2.8 | − 1.0 | + 1.8 | 17 | 17 | " |

Mean of Velocities = +5.1

*α Persei*

$\alpha = 3^h\ 17^m.2$ $\delta = +49°\ 31'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 922. | Oct. 29, 1904 | − 9.2 | − 9.5 | − 9.4 | +10.4 | + 1.0 | 14 | 14 | Maag |
| 928.... | " 30, " | − 9.4 | − 8.6 | − 9.0 | +10.0 | + 1.0 | 14 | 14 | " |
| 940.... | Nov. 6, " | − 6.4 | − 5.6 | − 6.0 | + 7.0 | + 1.0 | 16 | 16 | " |
| 943.. . | " 11, " | − 6.9 | − 6.5 | − 6.7 | + 4.9 | − 1.8 | 16 | 16 | " |
| 947.... | " 22, " | + 3.3 | + 0.8 | + 2.0 | − 0.1 | + 1.9 | 15 | 15 | " |

Mean of Velocities = +0.6

*α Tauri*

$\alpha = 4^h\ 30^m.2$ $\delta = +16°\ 18'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 949.... | Nov. 30, 1904 | +58.3 | +59.1 | +58.7 | − 0.1 | +58.6 | 19 | 19 | Maag |
| 951.... | Dec. 14, " | +60.6 | +61.4 | +61.0 | − 7.3 | +53.7 | 19 | 19 | " |
| 954.... | " 30, " | +70.4 | +68.4 | +69.4 | −15.1 | +54.3 | 16 | 16 | " |
| 957.... | " 31, " | +71.4 | +70.7 | +71.0 | −15.5 | +55.5 | 16 | 16 | " |
| 961.... | Jan. 1, 1905 | +73.1 | +73.5 | +73.3 | −15.9 | +57.4 | 18 | 18 | " |

Mean of Velocities = +55.9

TABLE V—*Continued*

$\alpha = 5^h\ 9^m.3$ — *α Aurigae* — $\delta = +45°\ 54'$

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 562.... | Jan. 22, 1899 | +57.1 | +55.7 | +56.4 | −18.7 | +37.7 | 15 | 16 | Lord |
| 563.... | " 25, " | +56.1 | +53.6 | +54.8 | −19.8 | +35.0 | 19 | 22 | " |
| 564.... | " 29, " | +51.9 | +47.5 | +49.7 | −21.1 | +28.6 | 19 | 20 | " |

Mean of Velocities = +33.8

$\alpha = 7^h\ 38^m.4$ — *κ Geminorum*[1] — $\delta = +24°\ 38'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 595.... | Jan. 21, 1900 | +27.1 | +25.9 | +26.5 | − 4.9 | +21.6 | 15 | 15 | Lord |
| 601.... | " 26, " | +28.9 | +27.0 | +28.0 | − 7.2 | +20.8 | 15 | 15 | " |
| 602.... | " 28, " | +33.4 | +30.2 | +31.8 | − 8.4 | +23.4 | 14 | 14 | " |
| 603.... | Feb. 13, " | +40.8 | +36.1 | +38.4 | −15.8 | +22.6 | 15 | 15 | " |
| 605.... | " 19, " | +40.2 | +36.8 | +38.5 | −18.4 | +20.1 | 15 | 15 | " |

Mean of Velocities = +21.7

$\alpha = 7^h\ 39^m.2$ — *β Geminorum* — $\delta = +28°\ 16'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 848.... | Mar. 23, 1904 | +31.4 | +32.3 | +31.8 | −27.6 | + 4.2 | 15 | 15 | Maag |
| 852.... | " 28, " | +32.6 | +29.8 | +31.2 | −28.3 | + 2.9 | 17 | 16 | " |
| 857.... | April 4, " | +35.0 | +33.7 | +34.4 | −29.0 | + 5.4 | 21 | 21 | " |
| 858.... | " 5, " | +38.3 | +36.4 | +37.4 | −29.1 | + 8.3 | 17 | 17 | " |
| 860.... | " 14, " | +34.3 | +35.1 | +34.7 | −29.2 | + 5.5 | 16 | 16 | " |

Mean of Velocities = +5.3

$\alpha = 8^h\ 22^m.0$ — *o Ursae Majoris*[2] — $\delta = +61°\ 3'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 604.... | Feb. 13, 1900 | +36.1 | +32.0 | +34.0 | −12.4 | +21.6 | 17 | 17 | Lord |
| 606.... | " 19, " | +36.9 | +35.0 | +36.0 | −14.3 | +21.7 | 17 | 17 | " |
| 611.... | Mar. 21, " | +44.2 | +40.5 | +42.4 | −21.1 | +21.3 | 15 | 16 | " |
| 613.... | " 22, " | +46.5 | +43.3 | +44.9 | −21.2 | +22.7 | 14 | 16 | " |
| 615.... | " 31, " | +43.3 | +39.5 | +41.4 | −22.5 | +18.9 | 16 | 15 | " |
| 617.... | Apr. 4, " | +46.1 | +43.3 | +44.7 | −22.7 | +22.0 | 14 | 16 | " |

Mean of Velocities = +21.4

$\alpha = 10^h\ 14^m.4$ — *γ Leonis* — $\delta = +20°\ 21'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 861.... | Apr. 14, 1904 | − 3.7 | − 7.6 | − 5.6 | −23.9 | −29.5 | 16 | 16 | Maag |
| 862.... | " 16, " | − 6.2 | − 9.4 | − 7.8 | −24.5 | −32.3 | 11 | 11 | " |
| 864.... | " 19, " | − 6.0 | −12.9 | − 9.4 | −25.2 | −34.6 | 17 | 17 | " |
| 868.... | May 5, " | − 2.5 | − 6.3 | − 4.4 | −28.1 | −32.5 | 16 | 16 | " |
| 870.... | " 9, " | − 3.5 | − 5.2 | − 4.4 | −28.4 | −32.8 | 15 | 15 | " |

Mean of Velocities = −32.3

[1] All spectrograms taken with two dense 60° prisms. Comparison *Fe* and *Hγ*. Reduction slightly different from that explained in paper.

[2] Spectrograms taken and reduced same as *κ Geminorum*.

TABLE V—*Continued*

$\alpha = 10^h\ 57^m.6$ *α Ursae Majoris* $\delta = +62°\ 17'$

| Plate No. | Date of Photograph | Observed Velocity | | | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| | | V. L. | V. R. | Mean | | | | | |
| 681.... | Apr. 20, 1902 | +15.9 | +11.0 | +13.4 | −18.5 | − 5.1 | 17 | 17 | Lord |
| 682.... | " 22, " | +14.3 | + 9.6 | +12.0 | −18.6 | − 6.6 | 19 | 19 | " |
| 683.... | " 23, " | +15.8 | +12.8 | +14.3 | −18.7 | − 4.4 | 16 | 18 | " |
| 687.... | " 24, " | +15.3 | +11.6 | +13.4 | −18.7 | − 5.3 | 19 | 19 | " |
| 690.... | " 27, " | +14.8 | +11.3 | +13.0 | −18.9 | − 5.9 | 16 | 16 | " |

Mean of Velocities = −5.5

$\alpha = 11^h\ 45^m.5$ *β Virginis* $\delta = +2°\ 20'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 696.... | Apr. 30, 1902 | +28.4 | +27.0 | +27.7 | −20.3 | + 7.4 | 19 | 19 | Lord |
| 706.... | May 8, " | +33.0 | +28.2 | +30.6 | −23.0 | + 7.6 | 18 | 18 | " |
| 709.... | " 9, " | +30.9 | +29.8 | +30.4 | −23.2 | + 7.2 | 16 | 16 | " |
| 716.... | " 26, " | +37.8 | +35.3 | +36.6 | −27.3 | + 9.3 | 19 | 19 | " |
| 717.... | " 28, " | +39.0 | +34.8 | +36.9 | −27.7 | + 9.2 | 21 | 21 | " |

Mean of Velocities = +8.1

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 696.... | Duplicate Measures | +28.6 | +28.8 | +28.7 | | + 8.4 | 18 | 18 | Maag |
| 706.... | | +30.5 | +29.6 | +30.0 | | + 7.0 | 16 | 15 | " |
| 709.... | | +31.8 | +32.0 | +31.9 | | + 8.7 | 18 | 18 | " |
| 716.... | | +35.4 | +34.0 | +34.7 | | + 7.4 | 19 | 19 | " |
| 717.... | | +36.3 | +37.7 | +37.0 | | + 9.3 | 18 | 18 | " |

Mean of Velocities = +8.2

$\alpha = 12^h\ 7^m.52$ *ε Virginis* $\delta = +11°\ 30'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 713.... | May 12, 1902 | + 7.6 | + 9.2 | + 8.4 | −19.0 | −10.6 | 17 | 17 | Maag |
| 719.... | June 9, " | +15.4 | +14.8 | +15.1 | −26.2 | −11.1 | 15 | 15 | " |
| 722.... | " 11, " | +19.0 | +19.4 | +19.2 | −26.4 | − 7.2 | 14 | 14 | " |
| 723.... | " 16, " | +15.6 | +14.1 | +14.8 | −26.9 | −12.1 | 19 | 19 | " |
| 725.... | " 21, " | +16.5 | +16.4 | +16.4 | −27.4 | −11.0 | 17 | 17 | " |

Mean of Velocities = −10.4

$\alpha = 13^h\ 49^m.9$ *η Boötis*[1] $\delta = +18°\ 54'$

| Plate No. | Date of Photograph | V. L. | V. R. | Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 572.... | May 13, 1899 | +30.0 | +24.2 | +27.1 | −14.6 | +12.5 | 11 | 10 | Lord |
| 581.... | " 24, " | +33.9 | +29.2 | +31.6 | −18.2 | +13.4 | 18 | 17 | " |
| 585.... | June 2, " | +34.3 | +33.9 | +34.1 | −20.4 | +13.7 | 11 | 10 | " |
| 586.... | " 10, " | +41.1 | +35.4 | +38.2 | −22.6 | +15.6 | 13 | 11 | " |
| 587.... | " 17, " | +40.6 | +33.3 | +37.0 | −23.9 | +13.1 | 13 | 12 | " |
| 588.... | " 18, " | +35.8 | +34.4 | +35.1 | −24.0 | +11.1 | 10 | 13 | " |
| 591.... | " 29, " | +40.0 | +36.5 | +38.2 | −25.3 | +12.9 | 13 | 13 | " |

Mean of Velocities = +13.2

[1] Variable discovered by Dr. Moore. *Lick Observatory Bulletin* No. 70; *Astrophysical Journal*, **19**, 246, 1904. See *η Boötis* on next page.

TABLE V—*Continued*

*η Boötis*

$\alpha = 13^h\ 49^m.9$ $\delta = +18°\ 54'$

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 685.... | Apr. 23, 1902 | +13.2 | +11.7 | +12.4 | − 6.3 | + 6.1 | 16 | 16 | Maag |
| 692.... | " 27, " | +18.0 | +18.1 | +18.0 | − 7.9 | +10.1 | 17 | 17 | " |
| 699.... | May 2, " | +20.5 | +19.5 | +20.0 | −10.0 | +10.0 | 14 | 14 | " |
| 707.... | " 8, " | +18.9 | +16.6 | +17.8 | −12.4 | + 5.4 | 11 | 11 | " |
| 710.... | " 9, " | +20.2 | +15.6 | +17.9 | −12.8 | + 5.1 | 12 | 12 | " |

Mean of Velocities = +7.3

*α Boötis*[1]

$\alpha = 14^h\ 11^m.1$ $\delta = +19°\ 42'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 686.... | Apr. 23, 1902 | + 2.2 | + 0.3 | + 1.2 | − 4.1 | − 2.9 | 19 | 14 | Maag |
| 700.... | May 2, " | + 5.2 | + 3.7 | + 4.4 | − 7.9 | − 3.5 | 15 | 15 | " |
| 711.... | " 9, " | + 6.7 | + 6.8 | + 6.8 | −10.7 | − 3.9 | 13 | 13 | " |
| {714... | " 12, " | +12.9 | +11.6 | +12.2 | −11.8 | + 0.4 | } 16 | 16 | " |
| {714... | Duplicate | +12.4 | +11.6 | +12.0 | −11.8 | + 0.2 | } 16 | 16 | " |
| 720.... | June 9, " | +17.7 | +17.2 | +17.4 | −21.2 | − 3.8 | 15 | 15 | " |
| 726.... | " 21, " | +21.6 | +19.0 | +20.3 | −23.7 | − 3.4 | 21 | 21 | " |
| 871.... | May 10, 1904 | + 8.4 | + 8.6 | + 8.5 | −11.0 | − 2.5 | 17 | 17 | " |
| 881.... | " 28, " | +11.0 | +12.2 | +11.6 | −17.3 | − 5.7 | 18 | 18 | " |

Mean of Velocities = −3.2

*ε Boötis*

$\alpha = 14^h\ 40^m.6$ $\delta = +27°\ 30'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 872.... | May 10, 1904 | − 7.7 | − 8.7 | − 8.2 | − 8.5 | −16.7 | 19 | 19 | Maag |
| 876.... | " 20, " | − 4.1 | − 7.0 | − 5.6 | −11.9 | −17.5 | 15 | 15 | " |
| 877.... | " 27, " | + 1.4 | + 1.1 | + 1.2 | −14.0 | −12.8 | 17 | 17 | " |
| 882.... | " 28, " | − 0.5 | − 3.7 | − 2.1 | −14.3 | −16.4 | 16 | 16 | " |
| 885.... | June 3, " | + 2.9 | + 0.5 | + 1.7 | −15.9 | −14.2 | 14 | 14 | " |

Mean of Velocities = −15.5

*α Serpentis*

$\alpha = 15^h\ 39^m.3$ $\delta = +6°\ 44'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 878.... | May 27, 1904 | +11.8 | +11.7 | +11.8 | − 6.8 | + 5.0 | 15 | 15 | Maag |
| 889.... | June 11, " | +15.8 | +16.7 | +16.2 | −13.0 | + 3.2 | 13 | 13 | " |
| 892.... | " 12, " | +23.2 | +20.4 | +21.8 | −13.4 | + 8.4 | 11 | 11 | " |
| 895.... | " 16, " | +23.6 | +20.5 | +22.0 | −15.0 | + 7.0 | 16 | 16 | " |
| 897.... | " 17, " | +21.7 | +21.0 | +21.4 | −15.4 | + 6.0 | 14 | 14 | " |

Mean of Velocities = +5.9

[1] Plate 714 rejected before measuring as being too poor to measure. Then later measured and reduced under a misunderstanding. Should be rejected.

TABLE V—*Continued*

*ζ Herculis*

$\alpha = 16^h\ 37^m.5$ $\delta = +31^\circ\ 47'$

| PLATE No. | DATE OF PHOTOGRAPH | OBSERVED VELOCITY | | | RED. TO ⊙ | RADIAL VELOCITY | NO. OF LINES, V. L. | NO. OF LINES, V. R. | MEASURED BY |
|---|---|---|---|---|---|---|---|---|---|
| | | V. L. | V. R. | Mean | | | | | |
| 879.... | May 27, 1904 | −66.4 | −67.2 | −66.8 | − 1.7 | −68.5 | 18 | 18 | Maag |
| 884.... | " 28, " | −67.4 | −67.6 | 67.5 | − 2.0 | −69.5 | 17 | 17 | " |
| 887.... | June 9, " | −67.5 | −66.4 | 67.0 | − 5.5 | −72.5 | 14 | 14 | " |
| 896.... | " 16, " | −64.9 | −67.1 | 66.0 | − 7.1 | −73.1 | 18 | 18 | " |
| 898.... | " 17, " | −60.1 | −59.7 | 59.9 | − 7.4 | −67.3 | 14 | 14 | " |

Mean of Velocities = −70.2

*β Draconis*

$\alpha = 17^h\ 28^m 2$ $\delta = +52^\circ\ 23'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 754.... | July 22, 1903 | −15.7 | −14.4 | −15.0 | − 5.0 | −20.0 | 16 | 16 | Maag |
| 758.... | " 24, " | −10.5 | −11.4 | −11.0 | − 5.6 | −16.6 | 15 | 15 | " |
| 761.... | " 31, " | −12.0 | −12.4 | −12.2 | − 6.4 | −18.6 | 16 | 16 | " |
| 764.... | Aug. 7, " | − 8.6 | − 9.5 | − 9.0 | − 6.9 | −15.9 | 17 | 17 | " |
| 766.... | " 9, " | − 9.1 | −10.1 | − 9.6 | − 6.8 | −16.4 | 15 | 15 | " |

Mean of Velocities = −17.5

*χ Draconis*

$\alpha = 18^h\ 22^m.9$ $\delta = +72^\circ\ 41'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 787.... | Sept. 1 1903 | +26.9 | +26.9 | +26.9 | + 3.2 | +30.1 | 12 | 12 | Maag |
| 790.... | " 2, " | +29.7 | +29.2 | +29.4 | + 3.2 | +32.6 | 12 | 12 | " |

Mean of Velocities = +31.4

*δ Draconis*

$\alpha = 19^h\ 12^m.5$ $\delta = +67^\circ\ 29'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 756.... | July 23, 1903 | +25.9 | +24.6 | +25.2 | + 3.5 | +28.7 | 17 | 17 | Maag |
| 765.... | Aug. 7, " | +26.6 | +26.1 | +26.4 | + 3.2 | +29.6 | 19 | 19 | " |
| 768.... | " 11, " | +24.2 | +23.6 | +23.9 | + 2.9 | +26.8 | 19 | 19 | " |
| 771.... | " 16, " | +24.5 | +22.6 | +23.6 | + 2.8 | +26.4 | 16 | 16 | " |
| 774.... | " 17, " | +24.7 | +21.9 | +23.3 | + 2.9 | +26.2 | 14 | 14 | " |

Mean of Velocities = +27.5

*β Cygni*

$\alpha = 19^h\ 26^m.7$ $\delta = +27^\circ\ 45'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 903.... | Sept. 10, 1904 | − 9.3 | −13.7 | −11.5 | −14.5 | −26.0 | 14 | 14 | Maag |
| 904.... | " 13, " | − 6.9 | − 6.8 | − 6.8 | −15.2 | −22.0 | 13 | 13 | " |
| 905.... | " 14, " | − 4.9 | − 6.4 | − 5.6 | −15.4 | −21.0 | 15 | 15 | " |
| 906.... | " 15, " | − 5.1 | − 4.9 | − 5.0 | −15.6 | −20.6 | 20 | 20 | " |
| 907.... | " 17, " | − 7.1 | − 7.4 | − 7.2 | −15.9 | −23.1 | 13 | 13 | " |

Mean of Velocities = −22.5

TABLE V—*Continued*

*γ Cygni*

$\alpha = 20^h\ 18^m.6$ $\delta = +39°\ 56'$

| Plate No. | Date of Photograph | Observed Velocity | | | Red. to ⊙ | Radial Velocity | No. of Lines, V. L. | No. of Lines, V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| | | V. L. | V. R. | Mean | | | | | |
| 782.... | Aug. 21, 1903 | − 2.1 | − 2.8 | − 2.4 | − 1.4 | − 3.8 | 16 | 16 | Maag |
| 785.... | " 23, " | − 1.6 | − 2.5 | − 2.0 | − 1.9 | − 3.9 | 14 | 14 | " |
| 788.... | Sept. 1, " | + 0.2 | ± 0.0 | + 0.1 | − 4.5 | − 4.4 | 17 | 17 | " |
| 792.... | " 6, " | − 0.6 | + 1.9 | + 0.6 | − 5.8 | − 5.2 | 13 | 13 | " |
| 794.... | " 12, " | + 4.0 | + 4.2 | + 4.1 | − 7.2 | − 3.1 | 19 | 19 | " |

Mean of Velocities = −4.1

*ε Cygni*

$\alpha = 20^h\ 42^m.2$ $\delta = +33°\ 36'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 770.... | Aug. 12, 1903 | −16.2 | −16.1 | −16.2 | + 2.1 | −14.1 | 17 | 16 | Maag |
| 772.... | " 16, " | −12.8 | −14.0 | −13.4 | + 1.1 | −12.3 | 13 | 13 | " |
| 775.... | " 17, " | −11.9 | −12.5 | −12.2 | + 0.5 | −11.7 | 15 | 15 | " |
| 780.... | " 20, " | −16.8 | −13.9 | −15.4 | − 0.2 | −15.6 | 18 | 18 | " |
| 783.... | " 21, " | −10.9 | −10.6 | −10.8 | − 0.4 | −11.2 | 18 | 18 | " |

Mean of Velocities = −13.0

*ε Pegasi*

$\alpha = 21^h\ 39^m.3$ $\delta = +9°\ 25'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 799.... | Sept. 24, 1903 | +24.1 | +23.4 | +23.8 | −14.3 | + 9.5 | 13 | 13 | Maag |
| 802.... | " 25, " | +22.4 | +22.8 | +22.6 | −14.6 | + 8.0 | 13 | 12 | " |
| 808.... | Oct. 18, " | +27.2 | +26.1 | +26.6 | −22.8 | + 3.8 | 17 | 17 | " |
| 811.... | " 19, " | +27.3 | +26.4 | +26.8 | −23.1 | + 3.7 | 16 | 16 | " |
| 812.... | " 24, " | +29.9 | +29.2 | +29.6 | −24.2 | + 5.4 | 13 | 13 | " |

Mean of Velocities = +6.1

*δ Cephei*[1]

$\alpha = 22^h\ 25^m.5$ $\delta = +57°\ 54'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 800.... | Sept. 24, 1903 | −27.0 | −28.6 | −27.8 | + 3.7 | −24.1 | 10 | 10 | Maag |
| 803.... | " 25, " | −10.4 | −12.1 | −11.2 | + 3.5 | − 7.7 | 19 | 20 | " |

*η Pegasi*

$\alpha = 22^h\ 38^m.3$ $\delta = +29°\ 42'$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 776.... | Aug. 17, 1903 | −11.2 | −13.0 | −12.1 | +11.9 | − 0.2 | 17 | 17 | Maag |
| 781.... | " 20, " | −10.5 | −11.2 | −10.8 | +10.8 | ± 0.0 | 18 | 18 | " |
| 784.... | " 21, " | −13.5 | −13.0 | −13.2 | +10.6 | − 2.6 | 17 | 17 | " |
| 789.... | Sept. 1, " | − 8.0 | − 8.4 | − 8.2 | + 6.3 | − 1.9 | 17 | 17 | " |
| 793.... | " 6, " | − 6.9 | − 9.0 | − 8.0 | + 4.4 | − 3.6 | 17 | 17 | " |

Mean of Velocities = −1.7

[1] Variable radial velocity discovered by Bélopolsky in 1898. Rediscovered here.

Here $m_{\text{sun}}$, $m_{\text{star}}$, and $\Delta m$ are respectively the values of $m$ for the given line given in the table of solar lines, the mean of the three settings on the star line, and the correction computed by the curve

$$\Delta m = x + by + cz \ .$$

$k$ is the constant to convert displacements into velocity. We have therefore

$$r_v = k\sqrt{(r_{\text{sun}})^2 + (r_{\text{star}})^2 + (r_{\Delta m})^2} \ .$$

To evaluate these several probable errors, Mr. Maag selected three plates—good, fair, and poor. On each plate he selected five representative lines and made twenty-five pointings on each. For each line he computed the probable error of a single pointing, the average of the fifteen being 0.68, the values lying between the limits 0.34 and 0.81 in thousandths of a millimeter. As each line was bisected three times, we may assume that $r_{\text{star}} = \pm 0.5$ km per sec. The solar lines were much sharper and were measured on two different days, but each line depended upon the difference of two sets of pointings. We shall therefore be safe if we assume $r_{\text{sun}} = \pm 0.5$ km per sec. The value of $r_{\Delta m}$ was computed from the residuals between the observed C. – O. and the computed C. – O. for the comparison lines for ten plates, using the measures, violet end left. The ten values range from 0.81 to 2.11 with a mean of 1.3. From this we find $r_v = 1.85$ km per sec., using $k = 1.25$, its value for the middle of the plate. On the average, there were from 15 to 17 lines measured on each plate, and each plate was measured in two positions, whence we find 0.3 km per sec. as the probable error of a single plate deduced in this way as against 1.88 by the first method. This difference can only be due to two causes, namely, the error of identification of the lines and what might be called the probable error of taking the photograph. The first of these is very hard to determine, and as an attempted solution I selected ten stars at random and derived the probable errors of a single line from their mean for that plate alone, using the measures violet end left. These range from 1.8 to 4.8, with a mean of about 3.0 km per sec. This gives $\pm 0.5$ km per sec. as the probable error of a single plate. The difference between this value and $\pm 0.3$ is due, I believe, to errors in identification. This is smaller than we should expect, which may be par-

*Venus*

| Plate No. | Date of Photograph | Observed Velocity V. L. | Observed Velocity V. R. | Observed Velocity Mean | Computed Velocity | C.–O. | Number of Lines V. L. | Number of Lines V. R. | Measured by |
|---|---|---|---|---|---|---|---|---|---|
| 651.... | Nov. 27, 1901 | − 9.8 | −12.1 | −11.0 | −13.2 | −2.2 | 21 | 22 | Lord |
| 660.... | Dec. 4, " | − 9.7 | −10.8 | −10.2 | −13.2 | −3.0 | 17 | 19 | " |
| 661.... | " 15, " | −10.9 | −13.1 | −12.0 | −13.1 | −1.1 | 20 | 20 | " |
| 666.... | " 30, " | −10.2 | −10.2 | −10.2 | −12.4 | −2.2 | 18 | 17 | " |
| 668.... | " 31, " | − 9.7 | −10.7 | −10.2 | −12.3 | −2.1 | 17 | 18 | " |
| 673.... | Jan. 1, 1902 | −10.9 | −13.5 | −12.2 | −12.0 | +0.2 | 20 | 20 | " |
| 676.... | " 6, " | − 8.0 | − 9.8 | − 8.9 | −11.9 | −3.0 | 19 | 19 | " |
| 689.... | Apr. 26, 1902 | +15.5 | +12.6 | +14.0 | +14.0 | ±0.0 | 20 | 21 | Lord |
| 695.... | " 29, " | +19.4 | +16.6 | +18.0 | +14.0 | −4.0 | 21 | 21 | " |
| 705.... | May 5, " | +18.4 | +14.4 | +16.4 | +13.9 | −2.5 | 14 | 13 | " |
| 651.... | Duplicate | −10.9 | −10.3 | −10.6 | −13.2 | −2.6 | 20 | 20 | Maag |
| 658.... | Nov. 30, 1901 | −10.6 | −10.8 | −10.7 | −13.3 | −2.6 | 15 | 15 | " |
| 660.... | Duplicate | − 8.4 | − 8.8 | − 8.6 | −13.2 | −4.6 | 20 | 20 | " |
| 661.... | " | − 9.8 | − 8.1 | − 9.0 | −13.1 | −4.1 | 20 | 20 | " |
| 666.... | " | − 9.0 | − 8.2 | − 8.6 | −12.4 | −3.8 | 19 | 19 | " |
| 676.... | " | − 7.1 | − 6.8 | − 7.0 | −11.8 | −4.8 | 13 | 13 | " |
| 689.... | Duplicate | +13.8 | +15.0 | +14.4 | +14.0 | −0.4 | 13 | 13 | Maag |
| 705.... | " | +17.3 | +16.4 | +16.8 | +13.9 | −2.9 | 17 | 17 | " |
| 731.... | Apr. 10, 1903 | − 7.6 | − 8.2 | − 7.9 | − 9.8 | −1.9 | 18 | 18 | Maag |
| 734.... | " 18, " | − 9.3 | −11.9 | −10.6 | −10.4 | +0.2 | 12 | 12 | Lord |
| 747.... | July 5, " | −18.2 | −18.0 | −18.1 | −13.9 | +4.2 | 18 | 18 | Maag |
| 748.... | " 6, " | −15.4 | −17.1 | −16.2 | −13.8 | +2.4 | 18 | 17 | " |
| 749.... | " 7, " | −12.2 | −14.2 | −13.2 | −13.9 | −0.7 | 15 | 15 | " |
| 751.... | " 8, " | −16.7 | −17.6 | −17.2 | −13.9 | +3.3 | 19 | 19 | " |
| 952.... | Dec. 30, 1904 | −11.2 | −11.3 | −11.2 | −11.7 | −0.5 | 18 | 18 | Maag |
| 955.... | " 31, " | −12.4 | −10.7 | −11.6 | −11.9 | −0.3 | 15 | 15 | " |
| 962.... | Jan. 16, 1905 | −10.5 | − 9.7 | −10.1 | −12.5 | −2.4 | 18 | 18 | " |
| 964.... | " 26, " | −11.1 | −12.5 | −11.8 | −12.7 | −0.9 | 20 | 20 | " |
| 965.... | Feb. 4, " | −10.4 | − 8.9 | − 9.6 | −13.0 | −3.4 | 17 | 17 | " |
| 966.... | " 7, " | −11.2 | −14.8 | −13.0 | −13.0 | ±0.0 | 16 | 16 | " |
| 967.... | " 10, " | −11.3 | −13.0 | −12.2 | −13.1 | −0.9 | 18 | 18 | " |
| 968.... | " 15, " | −12.7 | −12.2 | −12.4 | −13.1 | −0.7 | 16 | 16 | " |

tially explained by the close similarity between the spectra of most of the stars I have observed and that of the Sun. The balance, namely, $\sqrt{(1.88)^2-(0.5)^2}=1.80$, is, I believe, entirely due to the numerous sources of error in taking the photograph. It forms practically the entire source of error, and I believe it could be materi-

ally reduced with a modernized, constant-temperature, rigid form of mounting for the spectroscope.

The final test is of course the observed velocities of the planets. I give above the observations of *Venus* in the line of sight. The velocity of *Venus* was computed in accordance with Campbell's paper previously cited. It is to be noted that there is a persistent constancy in the sign of the residuals C.–O. I am utterly at a loss to account for it, and can explain it only by the much abused "personal equation." It is very small, only about 0.002 on the plate, and could, I believe, be applied as a constant correction to my velocities.

In conclusion I wish to express my indebtedness to my assistant, Mr. Maag, who has done nearly all the computation and measurement. The photographs were always taken by myself. I should like also to call attention to the fact that it has been possible, with this small dispersion, to secure measurable spectrograms of stars of the fifth photographic magnitude, with a telescope of only 12½ inches clear aperture, and that in a bad sky. I feel confident that a spectroscope built from the ground up for this purpose would materially reduce the errors. Now, if such an instrument could be attached to a three-foot reflector, I believe we could easily reach stars of the photographic magnitude seven and one-half. It would, however, be restricted as to type of spectra.

EMERSON McMILLIN OBSERVATORY,
Columbus, Ohio,
March 7, 1905.

# THE MOTION OF THE MATTER COMPOSING THE TAIL OF COMET 1903 IV, OBSERVED JULY 24, 1903

By R. JAEGERMANN

Professor E. E. Barnard[1] observed first a motion of the tail on three photographs of the comet 1903 IV, taken July 24, 1903; and both Mr. Sebastian Albrecht[2] and Mr. Roberts[3] have obtained the same results by photographic means. Professor H. Kreutz[4] has investigated carefully the photographs secured by Mr. Roberts. I learn from the Greenwich Observatory[5] that on the photograph obtained there on July 24 there is, on account of the large scale, no trace of that side of the piece separated from the tail which is nearer the nucleus. This piece lies on the forward side of the comet with reference to the direction of motion of the comet. Therefore there is left for comparison only the photograph taken by Mr. Smith[6] at the Yale Observatory. Owing to the fact that the time of exposure of this photograph cannot be ascertained, it must be omitted from the discussion.

The middle of the time of exposure has been considered as the epoch for the photographs of Roberts and Quénisset. For the photographs of Barnard, Curtiss, and Wallace we have adopted the beginning of the exposure increased by 30 minutes; this was done on account of the long duration of these exposures.

By a triangulation by means of neighboring comparison stars, the co-ordinates $\alpha$, $\delta$ of the particular end of the tail were divided on the plates of Roberts,[7] Quénisset, and the reprints of the photographs of Barnard, Curtiss, and Wallace as given in the *Astrophysical Journal* and the *Bulletin of the Lick Observatory*. The positions of the stars were taken from the catalogues of the *Astronomische Gesell-*

[1] *Astrophysical Journal*, **18**, 212, 1903.

[2] *Lick Observatory Bulletin* No. 52; *Astrophysical Journal*, **19**, 124, 1904.

[3] *Knowledge*, 26, 201, 1903.

[4] *A. N.*, **166**, 279, 1904.

[5] *Monthly Notices*, 64, 85, December 1903.

[6] *Popular Astronomy*, **11**, 519.

[7] I. Roberts' photograph was kindly furnished to the writer by Mrs. Dorothea Klumpke Roberts.

*schaft.* The co-ordinates $a_{\text{comet}}$, $\delta_{\text{comet}}$ of the nucleus have been computed from Perrine's[1] parabolic elements, and corrections to them have been applied from the observations of the comet as published in *Astronomische Nachrichten* and the *Bulletin of the Lick Observatory.* By means of the *Berliner Jahrbuch* we have computed the $a \odot$ and $\delta \odot$ which belong to the epoch. These are the results:

| Observatory | Observer | Time of Exposure | Epoch (G. M. T.) | $a$ (1903.0) | $\delta$ |
|---|---|---|---|---|---|
| Crowborough... | Roberts.... | $0^h\ 45^m$ | 1903 July 24.43821 | 205° 56′ 15″ | +64° 42′ 44″ |
| Nanterre........ | Quénisset.. | 1 0 | 24.47917 | 206 2 0 | 64 38 18 |
| Yerkes.......... | Barnard... | 2 37 | 24.64375 | 206 36 5 | 64 22 43 |
| Lick............ | Curtiss.... | 5 30 | 24.72917 | 206 59 41 | 64 13 56 |
| Yerkes.......... | Wallace.... | 2 30 | 24.77014 | 207 11 35 | 64 9 7 |

For the sake of completeness we have given the seconds of arc, although they must be considered quite uncertain. Indeed, it is quite difficult to identify on comet plates corresponding points and to give the correct epochs of observations.

The co-ordinates of the Sun and the comet, the distance ($\rho$) of the nucleus from the Earth, and finally the angular distance $s$ of the end of the tail ($a$, $\delta$) from the nucleus ($a_{\text{comet}}$, $\delta_{\text{comet}}$) are given in the following table:

| | $a \odot$ | $\delta \odot$ | $a$ Comet | $\delta$ Comet | Log $\rho$ | $s$ |
|---|---|---|---|---|---|---|
| R............ | 123° 1′ 49.″7 | +19° 59′ 10.″9 | 202° 21′ 28″ | +64° 34′ 56″ | 9.554282–10 | 1° 32′ 17″ |
| Q............ | 4 16.1 | 58 40.3 | 202 5 12 | 30 30 | 9.555200 | 1 41 57 |
| B............ | 14 4.0 | 56 37.1 | 201 1 15 | 12 36 | 9.558893 | 2 25 32 |
| C............ | 19 9.0 | 55 32.9 | 200 28 59 | 3 16 | 9 560812 | 2 50 39 |
| W............ | 21 35.3 | 55 2.1 | 200 13 45 | 63 58 46 | 9.561733–10 | 3 2 56 |

The second and third values of $s$ are slightly larger than those given by Mr. Albrecht, while the last two show agreement with those derived by Albrecht. These new values of $s$ vary more regularly per hour in the various intervals than those given by Albrecht, as may be seen from the following figures:

| | Interval of Time | $\delta s$ in $1^h$ | | Interval of Time | $\delta s$ in $1^h$ |
|---|---|---|---|---|---|
| R.–Q........ | $0^h\ 59^m$ | 0° 9.′9 | Q.–C....... | $6^h\ 0^m$ | 11.′4 |
| R.–B........ | 4 56 | 10.8 | Q.–W....... | 6 59 | 11.6 |
| R.–C........ | 6 59 | 11.2 | B.–C....... | 2 3 | 12.2 |
| R.–W........ | 7 58 | 11.3 | B.–W....... | 3 2 | 12.3 |
| Q.–B........ | 3 57 | 11.0 | C.–W....... | 0 59 | 12.5 |

[1] *Lick Observatory Bulletin* No. 47, 127.

This regular change in the above derived values of $s$ gives a certain justification to their existence. The same results are obtained on adopting the instant of beginning the exposure, as proposed by Albrecht. Bessel's auxiliary angles[1] $P$, $S$ ($P'$ need not be computed at all), as well as the position angle at the comet's nucleus ($p$) of the end of the tail, and $p_0$ of the prolonged radius vector, together with the auxiliary angles $u-P'$, $u_0-P'$ are:[2]

| | $P$ | $S$ | $p$ | $p_0$ | $u-P'$ | $u_0-P'$ |
|---|---|---|---|---|---|---|
| R.......... | 178° 37′ 33″ | 110° 54′ 57″ | 83° 31′ 58″ | 89° 20′ 57″ | 255° 58′ 57″ | 272° 1′ 31″ |
| Q.......... | 178 20 8 | 110 59 12 | 50 10 | 89 5 36 | 257 36 28 | 6 53 |
| B.......... | 177 11 32 | 111 16 0 | 30 26 | 88 5 19 | 259 55 53 | 28 14 |
| C.......... | 176 36 48 | 111 24 34 | 29 40 | 87 34 56 | 261 30 35 | 39 7 |
| W.......... | 176 20 23 | 111 28 39 | 83 37 47 | 87 20 35 | 262 38 0 | 272 44 18 |

The angle $T$ gives the perspective reduction, $\Delta$ stands for the distance between the end of the tail and nucleus measured in the plane of the comet, $\phi = u - u_0 = (u-P') - (u_0 - P')$ is the angle between $\Delta$ and the radius vector, $R$ is the radius vector of the end of the tail, and finally $\omega$ is the angle between $R$ and the axis of the orbit of the comet (negative if before the perihelion). Their values are as follows:[3]

| | $T$ | Log $\Delta$ | $\phi = u - u_0$ | Log $R$ | $\omega$ |
|---|---|---|---|---|---|
| R................ | 103° 4′ 37″ | 7.997362–10 | −16° 2′ 34″ | 9.976842–10 | −107° 8′ 6″ |
| Q................ | 101 33 30 | 8.038981 | 14 30 25 | 9.976937 | 107 5 54 |
| B................ | 99 22 36 | 8.194732 | 12 32 21 | 9.977564 | 106 54 30 |
| C................ | 97 54 2 | 8.264147 | 11 8 32 | 9.978025 | 106 49 16 |
| W................ | 96 51 9 | 8.294064–10 | −10 6 18 | 9.978263–10 | −106 47 21 |

The linear distances, $\Delta$ and $R$, of the end of the tail from the nucleus and from the Sun, grow in the same regular fashion that was noticed in $s$; they do not jump in the way they do in Albrecht's tables.[4] They are, in fact, as follows:

[1] Bessel's formulæ for computing these and other angles were first corrected by Th. Bredichin as early as 1862 in his work *On the Tails of Comets* (Moscow, 1862; in Russian). But they have remained almost unknown on account of the limited use of the language of the original.

[2] See, for instance, *Lick Observatory Bulletin* No. 42, p. 101; and No. 52, p. 167 or the *Astrophysical Journal*, **19**, 125, 1904.

[3] Professor Dr. Th. Bredichin's *Mechanische Untersuchungen über Cometenformen*, in systematischer Darstellung von R. Jaegermann (St. Petersburg, 1903; Voss Sortiment, Leipzig), pp. 305, 314.

[4] *Lick Observatory Bulletin* No. 52, p. 164.

| | $\delta\Delta$ in 1 Sec. | $\delta R$ in 1 Sec. | | $\delta\Delta$ in 1 Sec. | $\delta R$ in 1 Sec. |
|---|---|---|---|---|---|
| R.-Q. ........ | 42.2 km | 8.8 km | Q.-C. ....... | 51.4 km | 16.5 km |
| R.-B. ........ | 48.1 | 13.3 | Q.-W. ...... | 52.0 | 17.2 |
| R.-C. ........ | 50.1 | 15.4 | B.-C. ....... | 55.0 | 20.4 |
| R.-W. ....... | 50.8 | 16.2 | B.-W. ...... | 55.1 | 20.9 |
| Q.-B. ........ | 49.6 | 14.4 | C.-W. ...... | 55.3 | 22.0 |

$\delta\Delta$ and $\delta R$ for the interval $Q.-B.$ really belong in the third and not in the fifth line, and they therefore do not depart from the general regular behavior of the rest. The same holds true for $\delta s$ for the same interval. By a graphic representation of $R$ and $\omega$, it can easily be seen that that end of the tail which is ahead of the radius when prolonged has moved in $7^h\ 58^m$ over an arc which is convex with respect to the Sun. The length of the arc is 0.00658 astronomical units, from which follows a mean orbital velocity of 34.3 km per second. During this interval of time of $7^h\ 58^m$ the end of the tail moved away from the Sun with a velocity of 16.2 km in the direction of the radius thus prolonged, and it moved away from the nucleus (measured likewise along $R$) with a mean velocity $\delta\ (\Delta \cos \phi) = 51.2$ km. The nucleus approached the Sun during this time with a mean velocity $\delta r = \delta(\Delta \cos \phi) - \delta R = 35.0$ km, while its orbital velocity was 43.5 km.

From this it is evident that the end of the tail has been moving on an arc which is convex toward the Sun, and that proves indisputably that the matter constituting the tail has been under the influence of a repulsive force. The convex arc evidently is part of a hyperbola, convex toward the Sun. The fact that the end of the tail was on all of the five plates constantly far ahead of the prolongation of the radius vector proves, on the basis of the mechanical theory of comets' tails, that the tail-matter emanated with a considerable initial velocity $g$, i. e., at a negative angle $G$ to the radius vector.

Matter thrown out before the perihelion passage, as in the present case, will, according to the theory, first approach the Sun, then pass through its hyperbolic perihelion, and after that depart from the Sun continuously. The steady increase of $R$ shows that the end of the tail under investigation had passed through the hyperbolic perihelion before Roberts' photograph was taken. In obtaining the elements of the hyperbolic orbit we must satisfy these two conditions:

1. The perihelion distance $Q$ must be smaller, or at most, equal to the value of $R$. (Roberts).

2. The time of passage through the perihelion must have taken place before, or at most at, the time of Roberts' photograph.

To fulfil these conditions it was necessary to assume for the repulsive force, $1-\mu$, of the Sun a value greater than 60 units[1] (the unit being the gravitational force of the Sun), hence much larger than those which Bredichin has found, namely, 18 and 36. To obtain the proper values for $\omega$ it was, moreover, necessary to assume for $g$ (velocity of emanation) a value even larger than the value $g=0.34$, adopted by Bredichin for the comet of 1744 (for the time unit $\frac{1}{k}=58.13244$ mean solar days), or $g=10.1$ km per second. The values adopted are $1-\mu=89$ and $g=0.42=12.5$ km per second. The increase in the value of $g$ by 0.08 is quite reasonable, since the matter in question is five times as light as in the previous case.

The elements of the hyperbola, which is located in the plane of the comet's orbit (already communicated in *A. N.* 3978), have been derived under the following initial conditions: The matter of the end of the tail is thrown out at the moment $M_0=1903$ July 23.36362 G. M. T. with an initial velocity $g=0.42$ and an angle of projection $G=-21°\ 30'$, with respect to the radius vector. It is under the effective and constant force $\mu=-88.05$ (repulsive).

The elements thus derived are:

$$\log P=8.019672-10 \qquad \log A=9.673273-10$$
$$\log Q=9.976695-10 \qquad \psi=\ \ 8°\ 28'\ 29.''8$$
$$\log E=0.004768 \qquad \omega_\pi=-107\ \ 17\ \ 29.3$$
$$M_\pi=1903 \text{ July } 24.28684 \text{ G. M. T.}$$

($P$ = semi-parameter, $Q$ = perihelion distance, $E$ = eccentricity, $2A$ = major axis, $\psi$ = asymptotic angle, $\omega_\pi$ = angle between the axis of the hyperbola and that of the orbit of the comet, $M_\pi$ = time of passage through the hyperbolic perihelion.)

We compute $\log R$ and $\omega$ from these elements for the moments of observation and form the difference between computation and

[1] *Bulletin de l'Académie Impériale des Sciences de St. Pétersbourg*, 1904, janvier, XX, 44.

observation for both; the orbital velocities are computed in the same manner. Here is the table:

| | log $R$ | $\Delta$ log $R$ | $\omega$ | $\Delta\omega$ | $H$ |
|---|---|---|---|---|---|
| R......... | 9.976852–10 | +0.000 010 | $-107°\ 7'\ 56''$ | $+10''$ | 31.12 km per sec. |
| Q......... | 976945 | + 8 | 107 5 20 | +34 | 31.68 |
| B......... | 977552 | – 12 | 106 54 58 | –28 | 35.12 |
| C......... | 978009 | – 16 | 106 49 36 | –20 | 37.50 |
| W......... | 978263 | 00 | 106 47 1 | +20 | 38.76 |

Considering the difficulties involved in measuring photographs of comet tails, we may well conclude that the hyperbola, determined above, represents the observations satisfactorily. The theoretical velocities agree likewise sufficiently with the velocity 34.3 km as derived from the observations.

After the theoretically derived instant of emanation, $M_0$, the comet was observed on the same day, July 23, by Barnard and Curtiss, but nothing of the phenomenon of July 24 was to be seen; because the matter of the end of the tail (sent out from the nucleus at an angle of $G = -21\overset{\circ}{.}5$ to the radius vector when the nucleus had an anomaly $v_0 = -108°\ 15'\ 19''$ with an initial velocity of 55.2 km in the hyperbola) entered the comet's orbit at an angle of $3°\ 13\overset{'}{.}5$; it therefore moved almost in the same direction with respect to the Sun as the nucleus. It was not until five hours later, at the anomaly $-108°\ 2'\ 12''$ and with an orbital velocity already diminished to 46.7 km that it again emerged from the comet's orbit, and then still continued its motion toward the Sun in the neighborhood of the nucleus. Three and five hours, respectively, after this egression the comet was photographed by Barnard and by Curtiss with velocities of 42.3 and 38.8 km. The orbital velocity of the nucleus increased from 43 km for $M_0$ to 43.2 for the moment of Curtiss' plate. On account of the continuous motion of the tail, which was almost in the same direction as that of the nucleus, it was in close proximity $\Delta$ to the nucleus when photographed by Barnard and Curtiss. $\Delta$ was smaller than the radius of the nebulous envelope. From the computation of the hyperbola, we find $14.35\ \Delta_{Ba.}$ (July 23) $= \Delta_{Ba.}$ (July 24), (1 mm = 10′), as measured on the photographs. Considering the slightly smaller distance between Earth and comet on July 23,

the distance in question will be about 1.2 mm (1 mm = 8.'9). The diameter of the nebulous envelope on July 23 in the direction of the radius vector is 5 mm on Barnard's photograph, and 4 mm perpendicular to this direction. The end point of the tail, which is entirely separated from the nucleus, is therefore inside this envelope and invisible. The same holds true of the photograph of Curtiss on July 23. For July 23 $\Delta$ is 15.53 times smaller than the $\delta$ on C.'s plate for July 24. It amounts therefore to $s = 11.'0$, measured from the nucleus; this corresponds to 2.65 mm on the plate of July 23 (1 mm = 4.'2), while the diameter of the envelope in the direction of the radius vector on the same plate is 15 mm, and 11 mm perpendicular to this direction. Distinct traces are noticeable on the plates of both Barnard and Curtiss (of July 23) of another end of the tail, which was separated before the one in question—about July 22.4792 G. M. T.

The tail under consideration was therefore also photographed on July 23, although, of course, it had not yet the length which it gained on the following day. This agrees well with the theory of cometary forms. Indeed, a hyperbolic motion, especially with a large repulsive force, will produce in a short time an immense extension and dissipation of the matter constituting the tail, since matter separated from the comet at an earlier epoch which is farther away from the nucleus will have a much larger orbital velocity than the other. On July 25 the comet was photographed by Barnard and also by Curtiss. Both plates fail to show the end of the tail, since, as the theory shows, it was either outside of, or on the very edge of, the exposed part. Furthermore, it had now attained a velocity of 74.5 and 81.9 km, respectively, so that, on account of resulting wide dissipation in space, the action of the particles was too weak to produce an impress on the plate. The orbital velocity is increasing continuously, and its limiting value of 406.8 km is reached in infinity.

To afford a better view of the matter we have tabulated the quantities $R$, $\omega$, $\Delta$, $H$ of the tail and $r$, $v$, and $h$ (orbital velocity) for the nucleus. Besides, to facilitate a graphical construction, we have given the co-ordinates $\xi_{\text{comet}}$, $\eta_{\text{comet}}$ of the nucleus, and $\xi_0$, $\eta_0$ of the end of the tail. They are referred to a system of axes which is immovable and pertains to the epoch $M_0$. (In the table the designation "egress" is the egress out of the orbit of the comet.)

| | | 1903 G. M. T. | $r$ | $R$ | $v$ | $\omega$ | $h$ | $H$ |
|---|---|---|---|---|---|---|---|---|
| 1.. | Emanation | July 23.3636 | 0.96019 | 0.96019 | $-108°15'.3$ | $-108°15'.3$ | 43.0 | 55.2 km per sec. |
| 2.. | Egress | 23.5757 | 0.95592 | 0.95515 | $-108$ 4.2 | $-108$ 2.2 | 43.1 | 46.7 |
| 3.. | Barnard | 23.6972 | 0.95348 | 0.95285 | $-107$ 57.8 | $-107$ 54.6 | 43.1 | 42.3 |
| 4.. | Curtiss | 23.8036 | 0.95134 | 0.95118 | $-107$ 52.2 | $-107$ 48.0 | 43.2 | 38.8 |
| 5.. | Perihelion | 24.2868 | 0.94158 | 0.94775 | $-107$ 26.3 | $-107$ 17.5 | 43.4 | 30.2 |
| 6.. | Roberts | 24.4382 | 0.93852 | 0.94809 | $-107$ 18.1 | $-107$ 7.9 | 43.4 | 31.1 |
| 7.. | Quénisset | 24.4792 | 0.93769 | 0.94830 | $-107$ 15.8 | $-107$ 5.3 | 43.5 | 31.7 |
| 8.. | Barnard | 24.6438 | 0.93436 | 0.94963 | $-107$ 6.8 | $-106$ 55.0 | 43.5 | 35.1 |
| 9.. | Curtiss | 24.7292 | 0.93263 | 0.95062 | $-107$ 2.1 | $-106$ 49.6 | 43.6 | 37.5 |
| 10.. | Wallace | 24.7701 | 0.93180 | 0.95118 | $-106$ 59.8 | $-106$ 47.0 | 43.6 | 38.8 |
| 11.. | Barnard | 25.6633 | 0.91366 | 0.97528 | $-106$ 9.4 | $-105$ 52.2 | 44.0 | 74.5 |
| 12.. | Curtiss | 25.8334 | 0.91020 | 0.98241 | $-105$ 59.6 | $-105$ 42.1 | 44.1 | 81.9 |

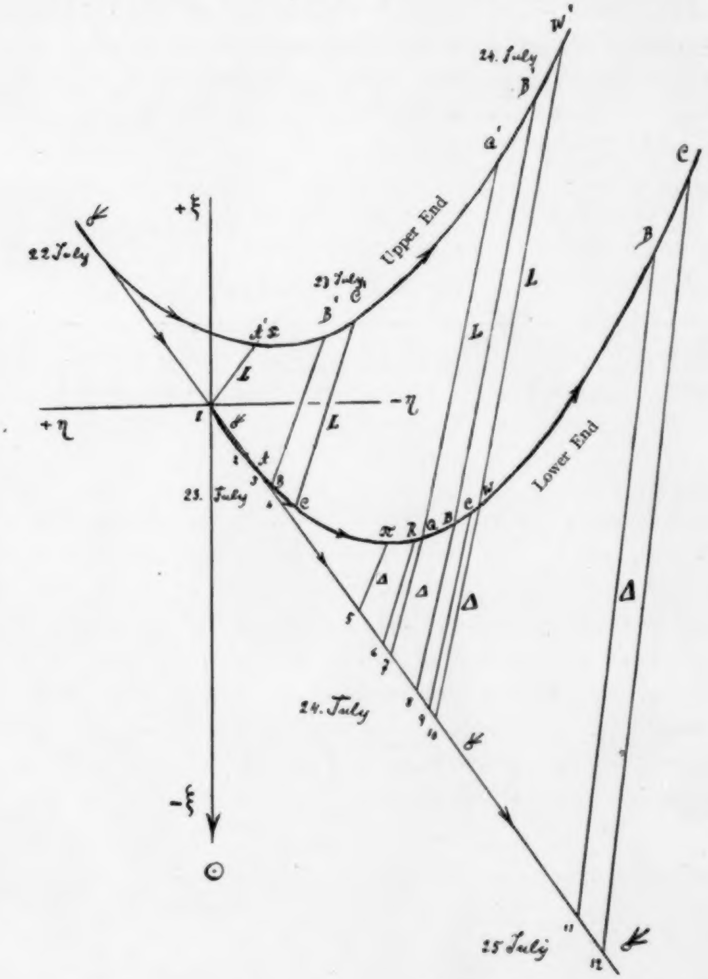
| | $\xi$ comet | $\eta$ comet | $\xi_0$ (obs.) | $\eta_0$ (obs.) | $\Delta$ (obs.) | $\xi_0$ (com.) | $\eta_0$ (com.) | $\Delta$ (com.) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | ......... | ......... | ....... |
| 2 | −0.00427 | −0.00309 | −0.00506 | −0.00365 | 0.00096 | ......... | ......... | ....... |
| 3 | −0.00673 | −0.00485 | −0.00736 | −0.00574 | 0.00109 | ......... | ......... | ....... |
| 4 | −0.00888 | −0.00640 | −0.00904 | −0.00757 | 0.00118 | ......... | ......... | ....... |
| 5 | −0.01871 | −0.01343 | −0.01257 | −0.01594 | 0.00663 | ......... | ......... | ....... |
| 6 | −0.02180 | −0.01563 | −0.01228 | −0.01859 | 0.00997 | −0.01230 | −0.01854 | 0.00994 |
| 7 | −0.02264 | −0.01623 | −0.01209 | −0.01930 | 0.01099 | −0.01211 | −0.01915 | 0.01094 |
| 8 | −0.02602 | −0.01862 | −0.01083 | −0.02220 | 0.01560 | −0.01081 | −0.02232 | 0.01566 |
| 9 | −0.02778 | −0.01986 | −0.00986 | −0.02370 | 0.01832 | −0.00983 | −0.02379 | 0.01837 |
| 10 | −0.02862 | −0.02046 | −0.00933 | −0.02443 | 0.01970 | −0.00932 | −0.02434 | 0.01968 |
| 11 | −0.04714 | −0.03344 | +0.01423 | −0.04061 | 0.06179 | ......... | ......... | ....... |
| 12 | −0.05070 | −0.03592 | +0.02124 | −0.04378 | 0.07237 | ......... | ......... | ....... |

The accompanying figure gives us an adequate idea (1 astronomical unit = 2000 mm) of the relative motion of the comet's tail with respect to the Sun, and also of the motion of the nucleus during this time. It will be noticed how well the computed arc represents the observed one, and it likewise becomes apparent why the end of the tail is invisible on the plates of July 23 and July 25, 1903.

Mr. S. A. Mitchell states in his article "Comet Borrelly and Light Pressure,"[1] that the velocity of the end of the tail on July 24 was 407 miles (= 655 km) with respect to the Sun. We have seen before that parts of the comet which are nearer to the nucleus will have an orbital velocity different from that of parts which are farther off. Although we may not speak of a velocity of the comet's tail, we may well speak of the velocities of different parts of it. On the other hand, such a velocity as Mr. Mitchell obtains is incomprehensible, since it is wholly incompatible with that velocity which we obtained for the end of the tail. Neither has the end of the tail farther away from the Sun the velocity attributed to it by Mr. Mitchell. Indeed, it is quite possible from the available plates to obtain approximately the orbit of the end farther away from the Sun which has been visible.

[1] *Astrophysical Journal*, **20**, 68, 1904.

The ejection of this matter took place July 22.4792 G. M. T. at an angle $G = -40°$ and initial velocity $g = 0.45$ (13.4 km). That $g$ and $G$ are in this case larger is amply indicated by Quénisset's plate, from




which plate we conclude that the separated tail has a considerable curvature (in the orbit of the comet), the concave side of which is turned forward (in the direction in which the nucleus is moving). The entire tail is ahead of the radius vector when prolonged ($\phi$ is counted negative for all its points) in such a manner that the end

farther away from the nucleus ($\Delta = 0.04585$, $\phi = 11^\circ\ 52'.8$) is about 3.5 times farther away from the prolongation of the radius vector than that end of the tail which is closer to the nucleus ($\Delta = 0.01094$, $\phi = -14^\circ\ 31'.1$). With due consideration for these and similar facts, the following hyperbolic elements have been obtained:

$M_0 = 1903$ July 22.4792 G. M. T. $\qquad M_\pi = 1903$ July 23.4035 G. M. T.

$1-\mu = 89.05$ $g = 0.45$ $G = -40^\circ$ $\qquad \log A = 9.680853-10$

$\log P = 8.134034-10$ $\qquad \psi = 9^\circ\ 33'\ 50''$

$\log Q = 9.984969-10$ $\qquad \omega_\pi = 107^\circ\ 56'\ 56''$

$\log E = 0.006079$

By means of this system we obtain:

| | 1903 G. M. T. | | $R$ | $\omega$ | $\xi_0$ | $\eta_0$ | $\Delta$ | $H$ |
|---|---|---|---|---|---|---|---|---|
| 1.. | Emanation | July 22.4792 | 0.97795 | $-109^\circ\ 0'.5$ | +0.01767 | +0.01287 | 0.00000 | 55.9 km per sec. |
| 2.. | [Emanation] | 23.3636 | 0.96601 | $-107\ \ 59.7$ | +0.00580 | −0.00439 | 0.00727 | 34.0 " |
| 3.. | Perihelion | 23.4035 | 0.96598 | $-107\ \ 56.9$ | +0.00577 | −0.00517 | 0.00800 | 33.9 " |
| 4.. | Barnard | 23.6972 | 0.96721 | $-107\ \ 36.6$ | +0.00695 | −0.01090 | 0.01496 | 36.8 " |
| 5.. | Curtiss | 23.8036 | 0.96825 | $-107\ \ 29.2$ | +0.00797 | −0.01299 | 0.01809 | 39.1 " |
| 6.. | Quénisset | 24.4792 | 0.98228 | $-106\ \ 43.1$ | +0.02173 | −0.02634 | 0.04551 | 61.9 " |
| 7.. | Barnard | 24.6438 | 0.98761 | $-106\ \ 32.1$ | +0.02697 | −0.02964 | 0.05412 | 68.5 " |
| 8.. | Wallace | 24.7701 | 0.99220 | $-106\ \ 23.8$ | +0.03148 | −0.03219 | 0.06124 | 73.6 " |

The moment 2 [emanation] refers to the end of the tail nearer to the nucleus, which took place about twenty-one hours after the first. The co-ordinates $\xi_0$, $\eta_0$ are given on the drawing, using the same scale; they are marked "upper end." From the diagram it is apparent that the length $L$ of the tail in which we have been interested is continuously increasing on account of the varying orbital velocities in the different parts of the tail. For convenience we have put together in tabular form the theoretical values of $\Delta$ and $H$ for both ends of the tail, and likewise the values for $L$:

| | 1903 G. M. T. | $\Delta$ | | $L$ | $H$ | |
|---|---|---|---|---|---|---|
| | | Beginning | End | | Beginning | End |
| [Emanation]...... | July 23.36 | 0.00000 | 0.00727 | 0.00727 | 55.2 km per sec. | 34.0 km per sec |
| Barnard ......... | 23.70 | 0.00109 | 0.01496 | 0.01490 | 42.3 " | 36.8 " |
| Curtiss.......... | 23.80 | 0.00118 | 0.01809 | 0.01800 | 38.8 " | 39.1 " |
| Quénisset........ | 24.48 | 0.01099 | 0.04551 | 0.03452 | 31.7 " | 61.9 " |
| Barnard.......... | 24.64 | 0.01560 | 0.05412 | 0.03847 | 35.1 " | 68.5 " |
| Wallace.......... | 24.77 | 0.01970 | 0.06124 | 0.04155 | 38.8 " | 73.6 " |

We can see from the following considerations that the second hyperbola represents approximately the motion of the more remote end of the tail. On both of the plates of Curtiss, dated July 23 and

24, the length of the visible tail (July 23) is almost equal to the distance between the tail end and nucleus (July 24); the same holds true for $L = 0.01800$ (July 23) and $\Delta = 0.01837$ (July 24). For the photograph of Quénisset (July 24) the theoretical value $\Delta = 0.0455$ agrees likewise with the observed value. On account of the continuous spreading out of the tail its light grows weaker and weaker, and this is clearly the case on the plates of Curtiss and Wallace on July 24.

On July 24 the mean velocity of the end nearer to the nucleus is 34 km, while the end farther away has one of 68. Averaging the two, we may call 51 km the mean velocity of the entire tail, but never 655 km!

Neither can the method by which Mr. Mitchell obtains a determination of $1-\mu$ furnish accurate results. He uses the angle $\phi$ for this purpose and derives separate values for $1-\mu$ for the months of June, July, and August. It is in itself an inaccurate proceeding, which suffers still more for values of the tail which are but slightly deflected from the radius vector, and more yet for values of the anomaly larger than 90°,— which indeed was the actual case in June and July. Employing the value zero for the initial velocity $g$, as Mr. Mitchell has done, his method is correct only for points on the axis of the conoid. Such points are generally not the ones which are being observed; it is rather the forward or the backward branch of the conoid which comes to be observed. Using the method mentioned above as Mr. Mitchell does, he must obtain a value for $1-\mu$ which in the first case is considerably larger, and in the second case considerably smaller, than that which holds true for the axis. But $1-\mu$ should have the same and constant value for all of the three cases. Even an emanation which takes place exactly in the direction toward the Sun will in larger measure pass over into the forward branch; and when $G$ has negative values, the forward branch may happen to be the only one which takes place (especially in cases where tails of the first type occur), while the backward branch may vanish to invisibility. By neglecting $g$ we obtain even for the central axis of the tail a value for $1-\mu$ which is considerably too large.

The existence of such a velocity of emanation has been proved by Bredichin's investigations, which include more than fifty comets; and in the case of Borrelly's comet it is further evidenced by the

*negative* value of $\phi$ on June 22 and 26, and on July 20 and 24. These angles cannot be accounted for by values of $1-\mu$, no matter how large, for $g=0$; even $1-\mu=\infty$ will for $g=0$ lead to a limiting value of $\phi=0$. This is the reason why Mr. Mitchell discards these observations, upholding his idea by the words: "It is impossible for the comet's tail to be ahead of the radius vector."

More accurate and concordant results might have been derived by Mr. Mitchell, if Albrecht's angle between tail and prolonged radius vector had not been employed. Strictly speaking, such angles do not exist; at most they indicate the initial or general direction of the tail; and that will not suffice for the accurate determination of $1-\mu$, especially with large values of the anomaly. That such angles do not exist can be seen from the fact that the tails have always a more or less marked curvature (after an accurate reduction upon the plane of the comet has been effected). To obtain more accurate values of $1-\mu$, it is necessary to select points of the tail which are as far as possible away from the nucleus, and from these we must determine values for $\phi$ and $\Delta$. For the same reason Curtiss' values of $\phi=u-u_0$ cannot be used for the determination of $1-\mu$ ("of the angle in the orbital plane of the comet between the radius vector from the Sun and the axis of the tail.")[1] Besides, as we have already stated, all these angles are not referred to the axis of the conoid of the tail, at least not the negative values.

On the plates of Roberts and Quénisset (July 24) there is another branch of the tail, between the tail which is separated from the nucleus and the radius vector when prolonged. There are two points, one near the middle, the other at the very end, of the branch for which $\phi=-5^\circ 14'.3$, $\Delta=0.01181$ and $\phi=-0^\circ 54'.1$ and $\Delta=0.02442$. For the representation of this branch it suffices to take $g=0.2$, $G=-8^\circ$, and $1-\mu=18$. The tail which is behind the radius vector belongs to type III. Its end points have for co-ordinates $\phi=+14^\circ\ 41'.4$, and $\Delta=0.02706$, $1-\mu=0.2$. Another streamer, which is bent still more, has for co-ordinates of the end point $\phi=+25^\circ\ 1'.4$, $\Delta=0.00676$; to it belongs $1-\mu=0.025$. The second type was not present. Still Mr. Mitchell finds one of this kind with $1-\mu=2.2794$, notwithstanding that the value of $\phi$ which enters into the computation is

[1] *Lick Observatory Bulletin* No. 42, p. 102.

$+18°$, i.e., much larger than the value of $\phi$ for the end point of the tail of type III. It is now quite easy to explain the values of $1-\mu$ for the first type. They vary without regularity from 3.5 to 114.9 during three months—a phenomenon not heretofore observed in any comet.

Mr. Mitchell says: "The light-pressure theory makes it plain why the angles between the radius vector and the tail continually increase up to the perihelion."[1] This well-known fact is simply a consequence of the motion of the nucleus on a conic and of the simultaneous motion of matter, emanating from the nucleus, and acted upon by a repulsive force located in the Sun which is inversely proportional to the square of the distance. From the mechanical phenomenon just related it is impossible to draw conclusions as to the physical nature of this force.

It is quite possible that the light-pressure as a repulsive force plays some important rôle in the formation of comets' tails, but in the case of Borrelly's comet it has not been proved that the light-pressure has acted in the sense of Arrhenius' theory, since the motion of the tail, investigated above, requires the assumption of a repulsive force sixty times greater than gravity. But this is what Mr. Mitchell has assumed. If we want to retain the hypothesis of light-pressure, we should have to maintain, on account of Schwarzschild's investigations, Bredichin's idea that the matter of the tail consists of gas molecules. These gas molecules, according to Lebedew,[2] are probably under the influence of a repulsive force exerted by the rays of the Sun, although it has not been possible to demonstrate this experimentally. The cause for the luminosity of comets' tails can thus be understood to be the fluorescence of highly illuminated gases, and this has been demonstrated experimentally by Lommel, Wiedemann, and Schmidt.

Repulsive forces were found to exist by Bredichin[3] in the case of Comet Rordame 1893 II ($1-\mu=36$), and by W. H. Pickering[4] in the case of Comet Swift ($1-\mu=39.5$). The existence of such forces proves the untenability of the light-pressure theory, from the stand-

[1] *Astrophysical Journal*, **20**, 67, 1904.

[2] *Physikalische Zeitschrift*, **4**, 17, 1902.

[3] *Bull. de l'Acad. Imp. des Sciences de St. Pétersbourg* T. II, 392, 1895.

[4] *Annals of Harvard College Observatory*, **32**, Part II, 1286.

point of the hypothesis of Arrhenius. Pickering's result is confirmed by preliminary investigations of the writer concerning the motion of the denser parts of the tail of Comet Swift 1892 I. They are carried out by means of the rigorous formulas for hyperbolic motion, and lead to the postulate that $1-\mu$ is certainly larger than 20.

On page 64 of Mr. Mitchell's paper there is this remark: "The electrical force, on which Bredichin explains his repulsions, has been shown by Lebedew not to have a sound physical basis." Against this statement this is to be said: Long before Lebedew, Bredichin admitted the possibility that the unknown repulsive energy of the Sun may well be of other than electrical character. In 1879 Bredichin expressed himself thus:

> J'emploie la dénomination de l'électricité pour l'énergie, qui émane du soleil et agit diversement sur les différents éléments chimiques des comètes, parceque cette dénomination est déjà introduite dans les théories physiques des comètes; mais il est bien possible, que les recherches ultérieures préciseront mieux la dénomination et les qualités de cette énergie.

Kepler guessed at the hypothesis of a light-pressure and Maxwell's famous investigations on the electromagnetic theory of light induced Fitzgerald, Lodge, and others to bring the light-pressure theory again into the foreground of scientific interest. It was then (1894) that Bredichin collected his ideas in the following statement:

> L'admission d'une charge électrique devient un peu risquant en égard aux nouveaux points de vue sur l'essence même de l'électricité, comformément aux quels la force répulsive électrique dans les queues sera peut-être regardée comme l'action répulsive des corps rayonnants.[1]

Moscow, December 29, 1904.
January 11, 1905.

[1] Bredichin–Jaegermann, pp. 483, 484.

# ON THE ENHANCED LINES OF IRON, TITANIUM AND NICKEL

By F. E. BAXANDALL

The results of a detailed study of the enhanced lines of iron, titanium, and nickel have recently been published by Dr. H. M. Reese.[1] In the case of each of the first two metals, he has compared the lines with those published by Sir Norman Lockyer,[2] and given rather lengthy lists of additional enhanced lines which do not appear in that record.

The importance of the enhanced lines of these metals in their relation to well-marked lines in certain types of stellar spectra has suggested an analysis of these extra lines being made, and an investigation of their authenticity as enhanced lines in the ordinary acceptation of the term.

## IRON

A table has been prepared which gives, in addition to Reese's spark and arc intensities, those of Exner and Haschek (spark) and Kayser and Runge (arc), extracted from Watts' *Index of Spectra.* In each case an intensity-range of 1 to 10 is used, so that the relative intensity in spark and arc ought to be roughly comparable. One would naturally expect, if Reese's extra lines are really enhanced, that Exner and Haschek's spark intensities would, in the majority of cases, be greater than those of Kayser and Runge, for the corresponding arc lines. So far is this from being the case, however, that only one of the sixty-five extra lines which can be compared in this way (five of Reese's lines are beyond Exner and Haschek's limits) fulfils that condition. This particular line (4303.34) was accidentally omitted from the Kensington record in preparing the paper for press, and has, since the publication of the enhanced line paper, been quoted in several Kensington publications.[3] There is no doubt whatever as to this being a genuine

[1] *Astrophysical Journal*, **19**, 322, 1904. [2] *Proc. R. S.*, **65**, 452, 1899.

[3] *Phil. Trans.*, A, **197**, 218, 1900. *Ibid.*, **201**, 218, 1904.

enhanced line, and it has its counterpart in many stellar spectra (notably *a Cygni*) and in the chromospheric and Fraunhoferic spectra.

With regard to the remaining sixty-four lines in Reese's record, this test, at any rate, tends to show that they cannot be accepted as enhanced lines. Such a test, however—the comparison of the published records of two different observers—is not so adequate and conclusive as a direct comparison of the spark and arc spectra. The identical negatives from which the Kensington record of enhanced lines was reduced have therefore been carefully re-examined, and the behavior in them of Reese's extra lines has been investigated. In the table given later, the last column is reserved for remarks on the occurrence and behavior of these lines in the Kensington arc and spark photographs.

Of the seventy lines, fifteen are stronger in arc than in spark, twenty-five are equally strong in both spectra, twenty do not occur in either spectrum, while six are slightly stronger in spark than arc, but are so nearly equal that one does not feel justified in recording them as enhanced lines: Three are outside the range of the Kensington grating photographs. The remaining one ($\lambda$ 4303.34), which is well enhanced, has previously been referred to and its absence from the published Kensington record explained.

Four of the most enhanced lines in Reese's extra list are $\lambda\lambda$ 4311.07 (2–3, tr.), 4322.92 (3, tr.), 4380.67 (3, tr.), and 4386.77 (3, tr.). There is not the slightest trace of these lines in any of the Kensington iron spectra, and none of them is recorded by Exner and Haschek.

It may be remarked that among Reese's extra lines such well-known strong spark and arc lines as the triplet $\lambda\lambda$ 4383.72, 4404.93, 4415.29 occur. These have been repeatedly used in Kensington publications as typical instances of the ordinary or unenhanced lines.

The inclusion by Reese of so many extra lines is probably due to the fact that he has selected a spark spectrum which has the majority of the lines slightly stronger than the corresponding lines in the arc. It would be possible, of course—to give an extreme case—so to arrange the relative exposures of the spark and arc photographs that all the spark lines might appear to be enhanced.

To be certain, however, of the lines being enhanced in the spark it is obviously best to give a slight bias to the arc lines in general; that is, to select the photographs so that the majority of lines are, if anything, slightly stronger in the arc; then any lines which are intrinsically stronger in the spark, after that preliminary condit'on has been fulfilled, may be legitimately accepted as enhanced lines.

Reese specially states that there is only one line stronger in his arc than in his spark photograph. This fact alone is nearly sufficient to show that he has, no doubt unconsciously, given a slight bias to the spark lines in general, so far as intensity is concerned, and explains the appearance in his record of so many extra lines. These are doubtless slightly stronger in his spark spectrum, but they cannot be unreservedly accepted as enhanced lines in the sense in which the term was first used.

There are five lines in Sir Norman Lockyer's list which Reese mentions as not appearing in his photographs. These are $\lambda\lambda$ 4302.35, 4451.75, 4462.30, 4541.40, 4635.40. A re-examination of the Kensington photographs shows that, with the possible exception of 4302.35 (whose arc and spark intensities are so nearly equal that it might have been better to omit it from the list), they are undoubtedly enhanced, $\lambda$ 4541.40 and 4635.40 especially being quite outstanding lines in the spark, and weak or lacking in the arc. Strangely enough, Reese quotes these two as being missing from his plates. They correspond to lines in the spectrum of $\alpha$ *Cygni*, similarly to the other well-enhanced iron lines, so that there is apparently no reason to doubt their authenticity.

*Reference to the plate.*—In the plate at the end of the paper, the spark and arc-spectra of iron are reproduced.

The photographs were taken with a four-prism Steinheil spectroscope, the large Spottiswoode coil being used to obtain the spark. The enhanced lines are conspicuously shown.

The lines marked L are some of the typical lines which appeared in Sir Norman Lockyer's list, and it will be seen that they are most distinctly stronger in the spark, although the majority of the lines are somewhat stronger in the arc.

The lines marked R are a few of those given by Reese in his

ANALYSIS OF REESE'S EXTRA ENHANCED *Fe* LINES

*a* = stronger in arc spectrum. *b* = equally strong in spark and arc. *c* = lacking both in arc and spark.

| Reese | | | Exner & Haschek (Spark) | Kayser & Runge (Arc) | Remarks on Behavior in Kensington Photographs |
|---|---|---|---|---|---|
| λ | Intensity | | | | |
| | Spark Max. 10 | Arc Max. 10 | Int. Max. 10 | Int. Max. 10 | |
| 4219.52 | 3–4 | 3 | 6 | 8 | Slightly stronger in spark |
| 4220.50 | 1 | tr | 2 | 4 | *b* |
| 4226.11 | 1 | tr | 1 | 4 | *a* |
| 4226.58 | 1 | tr | 1 | 4 | *a* |
| 4229.69 | tr | .. | 1 | 2 | Slightly stronger in spark |
| 4230.54 | tr | .. | .. | 1 | *c* |
| 4230.86 | tr | .. | .. | 1 | *c* |
| 4240.53 | 1 | .. | 1 | 2 | *b* |
| 4242.89 | tr | .. | 1 | 2 | *b* |
| 4246.24 | 1 | .. | 2 | 4 | *b* |
| 4250.96 | 6 | 5 | 8 | 10 | *b* |
| 4253.90 | 1 | .. | .. | 1 | *c* |
| 4268.94 | 3 | tr | 1 | 4 | *b* |
| 4271.94 | 7 | 6 | 10 | 10 | *b* |
| 4274.90 | 2 | 1 | .. | 2 | *a* ? *Cr* impurity |
| 4277.88 | 1 | .. | .. | 1 | *c* |
| 4279.60 | tr | .. | 1 | 1 | *c* |
| 4285.59 | 1–2 | 1 | 2 | 6 | Slightly stronger in spark |
| 4288.32 | tr | .. | 1 | 4 | *b* |
| 4290.50 | tr | .. | 1 | 2 | *b* |
| 4296.08 | tr | .. | .. | 1 | *c* |
| 4298.21 | 2 | 1 | 1 | 4 | *b* |
| 4303.34 | 1 | .. | 2 | 1 | Accidentally omitted from Kensington record in preparing paper for press |
| 4307.53 | 1 | .. | .. | .. | *c* |
| 4308.08 | 8 | 7 | 10 | 10 | *b* |
| 4309.20 | 1 | tr | 2 | 2 | *b* |
| 4310.28 | tr | .. | .. | .. | *c* |
| 4311.07 | 2–3 | tr | .. | 1 | *c* |
| 4322.92 | 3 | tr | .. | 1 | *c* |
| 4325.95 | 8 | 7 | 10 | 10 | *b* |
| 4328.95 | 2 | tr | .. | 1 | *c* |
| 4331.96 | 1 | .. | .. | 1 | *c* |
| 4343.45 | tr | .. | 1 | 2 | *b* |
| 4343.89 | tr | .. | 1 | 2 | *b* |
| 4346.73 | tr | .. | 1 | 4 | *b* |
| 4367.75 | 2 | 1–2 | 2 | 6 | *b* |
| 4371.52 | 1 | tr | .. | 1 | *c* |
| 4373.75 | tr | .. | 1 | 2 | *b* |
| 4380.67 | 3 | tr | .. | 1 | *c* |
| 4383.73 | 10 | 9 | 10 | 10 | *b* |
| 4386.77 | 3 | tr | .. | 1 | *c* |
| 4388.06 | 1 | tr | 2 | 4 | *b* |
| 4388.58 | 1–2 | 1 | 2 | 6 | *a* |
| 4390.05 | tr | .. | .. | 2 | *a* |

Analysis of Reese's Extra Enhanced *Fe* Lines—*Continued*

| Reese λ | Reese Intensity Spark Max. 10 | Reese Intensity Arc Max. 10 | Exner & Haschek (Spark) Int. Max. 10 | Kayser & Runge (Arc) Int. Max. 10 | Remarks on Behavior in Kensington Photographs |
|---|---|---|---|---|---|
| 4391.12...... | 1 | tr | 1 | 6 | *b* |
| 4392.68...... | tr | .. | 1 | 1 | *c* |
| 4401.46...... | 1 | tr | 1 | 6 | Slightly stronger in spark |
| 4404.93...... | 9 | 7–8 | 10 | 10 | *b* |
| 4407.99...... | 2–3 | .. | 2 | 6 | *b* |
| 4412.22...... | 2 | tr | .. | 2 | *c* |
| 4415.32...... | 7 | 6–7 | 8 | 10 | *b* |
| 4418.36...... | 2 | .. | .. | 1 | *c* |
| 4423.98...... | 1 | .. | .. | 1 | *c* |
| 4433.38...... | 2 | 1 | 2 | 6 | *a* |
| 4450.51...... | 1 | .. | 1 | 2 | Slightly stronger in spark |
| 4466.72...... | 4 | 3–4 | 5 | 8 | *b* |
| 4477.43...... | 1 | .. | .. | 1 | *c* |
| 4548.02...... | 1–2 | 1 | 2 | 8 | *b* |
| 4598.30...... | tr | .. | 1 | 6 | *a* |
| 4619.46...... | 1 | tr | 1 | 6 | *a* |
| 4637.69...... | 1 | tr | 1 | 6 | *a* |
| 4638.21...... | 1 | tr | 1 | 6 | *a* |
| 4669.34...... | tr | .. | 1 | 4 | *a* |
| 4673.36...... | tr | .. | 1 | 4 | *a* |
| 4691.59...... | 1 | tr | 1 | 6 | *a* |
| 4786.99...... | 1 | tr? | Beyond Exner & Haschek's range | 4 | *a* |
| 4789.83...... | 1–2 | tr | | 6 | *a* |
| 5002.05...... | 2 | 1 | | 8 | Beyond the range of the Kensington grating photographs |
| 5005.90...... | 2 | tr | | 6 | |
| 5006.31...... | 2 | 1–2 | | 8 | |

list of additional enhanced lines, but a glance at the plate will show that there is no indication of their being enhanced in the spark.

The two lines marked X are the Kensington lines which Reese says are missing from his plates. The absence of these two from his spectra is rather remarkable, especially as he seems to have photographed many lines which do not occur in the Kensington photographs at all.

### TITANIUM

In the case of titanium, Reese gives twenty-five lines in addition to those in the Kensington list, six of them, however, being outside the range of the latter. As to the remaining nineteen lines, reference to the photographs from which the Kensington reductions were made, failed to substantiate them as enhanced lines. A much

better and more extensive titanium spark photograph has been recently obtained, and an investigation of this shows that twelve of Reese's nineteen lines are stronger in the spark spectrum than in the arc, though most of these are very weak, even in the spark. Of the other seven, five are entirely lacking in both spark and arc spectra (three of these five are also missing from Exner and Haschek's record), one is appreciably stronger in arc than spark, and one is equally strong in spark and arc. All these are indicated in the accompanying table.

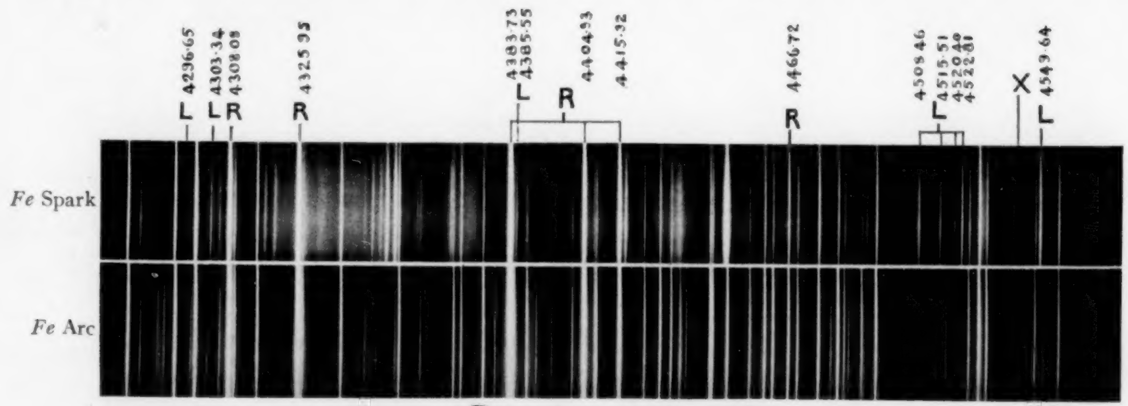
With regard to the Kensington line $\lambda$ 4308.60, which Reese says is probably identical with his $\lambda$ 4308.06, it may be said that the former published wave-length was an erroneous one. It should

ANALYSIS OF REESE'S EXTRA ENHANCED *Ti* LINES

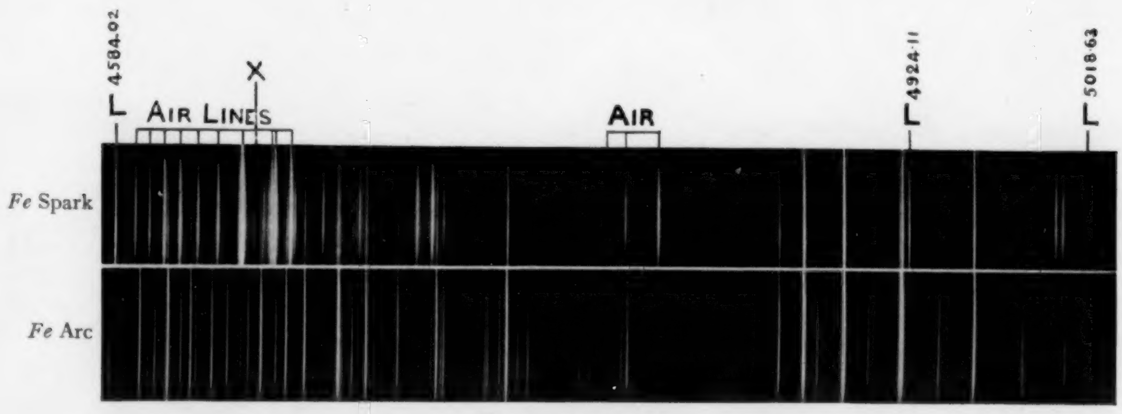
*a* = stronger in spark. *b* = lacking both in spark and arc.

| Reese | | | Exner & Haschek Intensity | | Remarks on Behavior in Kensington Photographs |
|---|---|---|---|---|---|
| $\lambda$ | Intensity | | | | |
| | Spark Max. 10 | Arc Max. 10 | Spark Mx. 100 | Arc Max. 20 | |
| 4288.04 | 3 | 2 | 2 | 1 | *a* |
| 4305.07 | 3 | 2 | .. | .. | *b* |
| 4337.56 | 2–3 | tr | 1 | .. | *a* |
| 4398.24 | 2–3 | 2 | .. | .. | *b* |
| 4398.45 | 2 | 1 | 1 | .. | *a* |
| 4409.40 | 2–3 | 1 | 1 | .. | *a* |
| 4409.68 | 2–3 | 1–2 | 1 | .. | *a* |
| 4412.09 | 2–3 | 1 | 1 | .. | *a* |
| 4418.50 | 4 | 3 | 2 | .. | *a* |
| 4432.27 | 2 | tr | 1 | .. | *b* |
| 4440.88 | 2 | 1 | .. | .. | *b* |
| 4441.89 | 3 | 2 | .. | .. | *a* |
| 4444.72 | 3 | 2 | 1 | .. | *a* |
| 4456.81 | 2 | .. | 1 | .. | *a* |
| 4471.02 | 3 | 2 | 2 | 1 | *a* |
| 4537.35 | 4 | 3 | 1 | 1 | Stronger in arc |
| 4544.18 | 3 | 2 | 1 | .. | Equally strong in spark and arc |
| 4568.49 | 2–3 | 1 | 1 | .. | *a* |
| 4583.59 | 2 | 1 | 1 | .. | *b* |
| | | | | | These lines are outside the limits of |
| 4609.55 | 4 | 3 | 1 | 1 | 1. those published by Sir Norman |
| 4657.37 | 3 | 1 | 1 | .. | 2. Lockyer. Reference to a more |
| 4687.98 | 2 | 1 | 1 | .. | 3. recent and unpublished Kensington reduction shows that Nos. 2, |
| 4764.08 | 4 | 2 | Beyond Exner & Haschek's range | | 4. 4, 5, and 6 are certainly enhanced |
| 4805.26 | 4 | 2 | | | 5. Nos. 1 and 3 are not enhanced in |
| 4911.39 | 5 | 1 | | | 6. the Kensington photographs |

PLATE XVI



Region 4275-4580



Region 4580-5030

have been λ 4308.10, the mistake probably being due to a transcriber's error in copying the paper for press. The corrected wave-length has been used in subsequent Kensington publications.[1]

### NICKEL

This metal was not one of those for which a record of enhanced lines was given in Sir Norman Lockyer's publication. The enhanced nickel lines have, however, since been reduced, as well as those of many other metals, and will be included in a future publication. The Kensington list has been compared with that given by Reese for the same element. Of the lines in the latter list λλ 4244.94, 4279.36, 4362.28, and 4509.42 are enhanced in the Kensington photographs, but there is no indication of the enhancement of the lines at λλ 4231.22, 4297.15, 4298.70, 4307.05, 4368.49, and 4398.66. It is only fair to say, however, that the enhancement of these in Reese's photographs is according to his spark and arc intensities, only very slight.

For each of the metals named Reese gives a further list of additional lines which he has not been able to find in any of the published records relating to the same metals. In view of this fact, it is extremely unlikely that they are genuine lines of the metals specified, and it has not been thought worth while to analyze them in the same way as the lines which have previously been recorded by other observers.

I must express my indebtedness to Sir Norman Lockyer for permission to use the excellent photographs involved in the discussion, and to Mr. C. P. Butler, who obtained them; some of these were taken with a large concave Rowland grating, and those reproduced in the plate with a four-prism Steinheil spectroscope.

SOLAR PHYSICS OBSERVATORY,
SOUTH KENSINGTON,
March 7, 1905

[1] *Phil. Trans.*, A, **197**, 218, 1900; *ibid.*, A, **201**, 218, 1904.

# NOTE ON THE CONDITIONS ATTENDING THE APPEARANCE OF THE ARGON LINES IN AIR

By A. S. King[1]

The spectrum of argon in gas mixtures was shown by Collie and Ramsay[2] to be not especially sensitive when a Geissler tube containing the mixture was excited by the ordinary glow-discharge, 37 per cent. of argon in nitrogen being required to show the argon spectrum. This was correct, however, only for the experimental conditions used by these observers, as Crookes[3] had previously shown that the argon in the air (less then 1 per cent.) would show its spectrum when atmospheric nitrogen was subjected to long-continued discharge in a tube with platinum electrodes, the nitrogen being for the most part removed by the electrodes. Newell[4] showed the same for ordinary air when the nitrogen was removed by passing a discharge through it in the presence of sulphuric acid and hydrogen or water vapor, the pressure in the tube becoming very low thereby. Further, Hartley[5] found that the spectrum given by copper, aluminium, or platinum electrodes in open air showed a number of lines which agreed very closely with lines in the argon spectrum and did not appear to belong to oxygen or nitrogen.

Lilienfeld showed in some recent experiments[6] that when an unusual discharge arrangement was used with tubes having outside electrodes and containing air at a pressure as high as 30 mm, the blue spectrum of argon appeared with the line spectrum of air. This gave promise of being a method of increasing the sensitiveness of gaseous spectra in general, and at the suggestion of Professor Warburg I undertook some experiments to determine the essential features of the spark-discharge needed to give the argon spectrum this degree of sensitiveness.

The discharge circuit used by Lilienfeld was first tried, and is

[1] Research Assistant of the Carnegie Institution of Washington.

[2] *Proc. R. S.*, **59**, 275, 1896.

[3] *Ibid*, **57**, 287, 1895.

[4] *Ibid.*, **57**, 346, 1895.

[5] *Ibid.*, **57**, 293, 1895.

[6] *Sitzungsberichte der K. Akademie der Wiss. zu Berlin*, 1904, p. 1196.

shown in Fig. 1. The terminals of an induction coil are connected to a battery of two or three Leyden jars in cascade which discharge across a spark-gap and through a self-induction spool of thick copper wire. In parallel with part or all of this self-induction is a tube having outside electrodes of tinfoil and containing air at about 3 mm pressure. The induction coil used gave a spark of about 25 cm in air, and was driven by either a Wehnelt or a mercury turbine interrupter. The currents used varied from 15 to 25 amperes and were supplied by a 110-volt dynamo circuit.

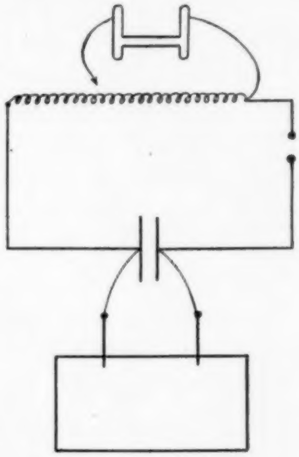


FIG. 1

It will be seen that this arrangement allows of considerable variation of the discharge through the tube, this discharge depending not only on the action of the Ruhmkorff and the capacity used, but on the amount of self-induction with which the tube is in parallel. The condenser gives an oscillating discharge, and a variable amount of this oscillating E. M. F. may be thrown on the electrodes of the tube. Lilienfeld found that with a small capacity a discharge could be maintained through the tube when in parallel with 20–30 turns of the spool, that then the line spectrum of air appeared, and a number of argon lines could be detected, the discharge producing but slight heating of the tube. This result was confirmed by the writer. Two Leyden jars, each 10 cm in diameter and with coating 13 cm high, were connected in cascade, and a spark-gap 2–2½ cm long in air was used. The self-induction spool had turns 3.5 cm in diameter and wound with two turns to the centimeter. The spectroscope had five Rutherfurd prisms and gave a large dispersion. To identify the argon lines, a Geissler tube containing very pure argon was placed horizontally in front of the slit, as in the experiment of Lilienfeld, so that its spectrum and that of the air-tube showed side by side. This arrangement has advantages over the use of a reflecting prism, but error must be guarded against if the slit is wide or the lines very bright, as then the slight illumination of the whole slit by the hori-

zontal tube suffices to give extensions of the lines which might be taken for lines in the other spectrum. In this case so narrow a slit was used that no extensions were visible. The Geissler tube was driven by a small coil, and a Leyden jar and spark-gap could be adjusted to give either the red or the blue spectrum.

The argon lines given by the air-tube did not develop into distinctness until the discharge had passed for a minute or more, as was observed by Lilienfeld; but after this period a number of the stronger argon lines could be seen, those most favorable for visual observation being in the blue-green at $\lambda\lambda$ 4880, 4848, 4806, 4765, 4736, 4727. The simple apparatus and low capacity, with the small heating of the tube, were favorable features of the arrangement.

The apparent small mean current-density in the tube, as indicated by the slight heating, combined with the appearance of the line spectrum of air, pointed to the conclusion that the wire of the inductance spool carries almost the whole current, while the rapid oscillations produce momentary high values of the current in the tube, which give the stimulus needed to bring out the spectrum of a very small amount of a gas.

A series of modifications of the spark-discharge was then made by the writer. The tube with outside electrodes was connected directly in series with the spark-gap. The discharge passed, with the spark-gap slightly shorter than before, but the air-bands now appeared, the same primary current and small capacity being used as before. By increasing the primary current the discharge could be forced into giving a fairly pure line spectrum, but the argon lines did not appear with this spectroscope, in which intensity was sacrificed to dispersion. The arrangement was, at any rate, not so favorable as that of Lilienfeld when so small a capacity was used. But the connection with the tube in series with the spark-gap seemed to give a greater mean current-density, as indicated by the heating of the capillary and brightness of the discharge.

A large capacity was then used for a series of trials. Two jars, each 19 cm in diameter and coated to a height of 36 cm, were connected in cascade, giving a capacity very large compared to that of the vacuum tube. While the results can be easily explained in accordance with those of the previous experiment, the arrangement of Lilienfeld proved now not to be the most advantageous.

The greater quantity of electricity when large capacity was used caused a heating of the capillary when the tube was in parallel with self-induction, so that the 0.3 mm capillary employed in the previous trials could not be used. An end-on tube with capillary 0.5 mm in diameter was found to give the best results. With this discharge the impedance offered by the inductance spool of negligible resistance is such that a discharge passes through the vacuum tube when in parallel with only a half-dozen turns of the spool, with a spark-gap of 2.5 cm. With so few turns the band spectrum of air appears, and the tube discharge must be strengthened with more self-induction in parallel until the line spectrum appears alone. When this condition is reached, the current in the tube branch is such that the capillary becomes quickly hot and can be used with safety but a few seconds at a time. A number of argon lines now appeared distinctly without requiring an interval to develop. Those mentioned before and $\lambda$ 4610 were the most conspicuous, while all strong lines in this region not too close to air lines could be readily perceived. The end-on tube adds greatly to the strength of the spectrum, but with this heavy discharge gives a strong continuous ground. Variations in the circuit were made to bring out the argon lines as well as possible. They were found not to be very sensitive, but appeared best with the strongest discharge that the tube would stand, while any approach to the band spectrum caused them to disappear.

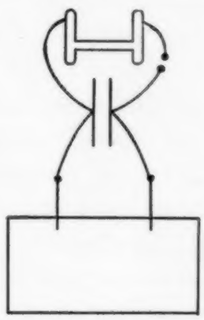


Fig. 2

The tube, with pressure unchanged, was then connected directly in series with the spark-gap (Fig. 2), and this was found with this capacity to give better results. The large capacity was able to maintain a discharge that was probably oscillatory, and with the turbine interrupter gave a thick, noisy spark 14 mm long between the knobs of the spark-gap in series with the tube. A bright discharge passed through the tube, and the two drawbacks of the arrangement with the tube in parallel, the heating of the capillary and the continuous spectrum, were now done away with. The spectrum showed the argon lines very distinctly, fully as strong as under the most favor-

able conditions of the former discharge; while the continuous spectrum was now so weak that a number of weaker argon lines could be recognized which were previously concealed. The heating of the tube was now comparatively slight, the capillary becoming only moderately warm after a run of three minutes. This small heating effect and the weakness of the continuous spectrum indicate a considerably smaller mean current-density in the tube when it is simply in series with the spark-gap. It was shown, however, that the difference of potential between the coatings of the tube was now considerably greater than with the tube in parallel with the self-induction. This could hardly be stated without a test, as the spark-gap is so much longer with the inductance spool that there is more energy in the discharge circuit to draw from, though the whole E. M. F. is not on the tube. The difference of potential in the two cases was roughly tested by an adjustable spark-gap connected to the two coatings of the tube; and the spark passed continuously at a distance about 35 per cent. greater when the tube was in series.

With the arrangement of Lilienfeld, the inductance spool necessarily has considerable influence on the discharge of the jars; and to retain this effect the spool should be kept in circuit with the other arrangement, the tube merely being put in series with the spool instead of in parallel. When this was done, the spectrum was little altered from the condition with no spool, but the argon lines were not so distinct, due probably to an approach to the band spectrum of air, which is the well-known effect of self-induction in the discharge circuit.

The substitution of a Wehnelt interrupter for the turbine showed the same relation between the two arrangements of the spark circuit, but it was necessary to use a shorter spark-gap, and the lines were correspondingly weakened.

A tube with inside electrodes of aluminium and containing air at about 3 mm pressure was next tried in series with the spark-gap. When the condensed discharge was made strong enough to give the line spectrum alone, the argon lines were even brighter with this tube than with that having outside electrodes. The inside electrodes would, of course, have disadvantages in general work.

In addition, the observation of Hartley[1] was fully confirmed by

[1] *Loc. cit.*

obtaining the argon lines by the spark in open air between copper electrodes, by which a strong air spectrum is given. There was no difficulty in identifying a number of argon lines by the eye. A photograph of this spectrum was made with the aid of a prism-spectrograph kindly lent me by Dr. Kreusler, the spectrum from the argon Geissler tube being photographed beside that of the spark in air. The argon lines in the blue and green sufficiently separated from air lines were given distinctly on this plate.

A still better comparison was made by examining some photographs which I took of the spectrum given by copper electrodes in air with a one-meter Rowland grating in the University of Bonn. The dispersion and definition of these photographs was such as to allow of a very accurate identification. On several plates taken with a powerful discharge from a large Klingelfuess inductor and a condensed spark 1 cm long between copper electrodes in air, the argon lines appeared very clearly, and the identification was rendered still more certain by comparison with a negative of the blue spectrum of argon in a vacuum tube, taken with the same grating and lent to me by Dr. Konen, of Bonn. The following argon lines could be identified with certainty in the air spectrum: $\lambda\lambda$ 4880, 4848, 4806, 4765, 4736, 4727, 4658, 4610, 4579, 4545, 4426, 4401, 4379, 4278. Other argon lines in this region occurring so near air lines as to make their appearance in the air spectrum uncertain are $\lambda\lambda$ 4590, 4503, 4430, 4371, 4348, 4331, 4228, 4130, 4104. Some differences appear in the relative intensities of lines in the two spectra, $\lambda$ 4658 being relatively weak in air and $\lambda\lambda$ 4482, 4266 either absent or extremely weak. Some differences of this sort would be expected from the very different discharges in the two cases.

The photographs made in Bonn show the argon lines best under the most powerful discharge conditions which I was able to produce, i. e., when the copper electrodes were at least 1 cm apart in air, with a thick, noisy spark. They appeared when either the middle of this spark or the region next to one pole was projected on the slit, somewhat stronger in the latter case. Visual observations pointing in the same direction were made during the present investigation by varying the separation of the electrodes in air. The lines were weak in the spark about 3 mm long, in which the discharge appeared almost

like an arc and was accompanied by rapid heating of the electrodes, as compared with the long and very noisy spark. While the current is, of course, different in these two cases, there can be little question that as the spark is lengthened and the discharge becomes more crackling the potential-gradient increases considerably faster than the current.

Summarizing the experimental results of Lilienfeld and myself, the essential condition to bring out the argon spectrum from very small quantities of the gas seems to be a high momentary value of the current-intensity produced by the conditions attending the oscillating discharge, by which the line spectrum of the air or other gas is given. Such a discharge, both in the experiments with vacuum tubes and with the spark in air, proved more favorable than the discharge in which a large mean current-density was given, but in which the value of the current is probably more uniform. In other words, the discharge must be such as to give for an exceedingly brief time a current-strength that could not be used continuously, and in this way give the greatest possible stimulus to the gas particles.

Without a more exact knowledge of the character of the spark-discharge under different circumstances, it would be mere speculation to say more than this, the purpose having been to use the argon spectrum as a test for the best conditions to make a very small percentage of a gas spectroscopically visible.

I wish to express my thanks to Professor Warburg for many helpful suggestions during these experiments.

PHYSICAL INSTITUTE, UNIVERSITY OF BERLIN,
March 1905.

# OBSERVATIONS WITH THE RUMFORD SPECTROHELIOGRAPH

By PHILIP FOX

The work with the Rumford spectroheliograph has been continued, with the aid of a grant from the Carnegie Institution of Washington to Professor Hale, throughout the year 1904, with no interruptions other than those caused by cloudiness. No changes have been made in the instrument since it was described in Volume III, Part I, of the *Publications of the Yerkes Observatory*. The general program of observations outlined there has been followed quite closely. On each clear day two plates were taken on the center of the H line, called $H_2$, and one on the edge of the H line toward the violet, called $H_1$. After these plates, which serve as a record, were secured, the observations were of a diversified character. Series of plates were taken with settings on different positions approaching the H line, ranging from $\lambda$ 3952 to $\lambda$ 3968.6, the object being to obtain more evidence on the question of levels. Many plates were obtained with increased dispersion on lines of the spectrum other than H. Prominence plates were taken with regularity during the latter part of the summer and through the autumn.

The series of $H_2$ plates shows a decided increase in activity over those of the year 1903. While there were few disturbed areas equal to that about the great spot of October 1903, every plate shows several smaller groups of flocculi, either associated with spots or detached. Nearly all of these plates have circular images, and they are being measured for a new determination of the solar rotation period at the height above the photosphere of the high-level flocculi. This determination will supplement the preliminary work on the Kenwood spectroheliograms, soon to be published by the Carnegie Institution. The same method of measurement, that of projecting the plates upon an adjustable ruled globe by means of an arc lamp and suitable lenses, is being employed.

The series of plates taken with different settings gradually approaching the center of the H line are of interest because they bear on the question of levels and the expansion of calcium vapor as it rises above the general mass of condensed vapors. Exposures with five or six different settings are made on one plate by moving the second slit, as a whole, by means of a micrometric screw. Often, however, when it is desired that the approach to the center of the line should be more gradual, and consequently more exposures are needed, the series is continued on a second plate. In this way series of photographs of some special feature have been made in rapid succession with ten or twelve different settings from $\lambda$ 3952.4 to $\lambda$ 3968.6. The time between successive exposures is just long enough to run the telescope back to the original declination so as to drive the feature across the first slit again, to move the second slit to the desired setting, and to change its width as approach is made to the more intense part of the dark absorption band. The records with settings on the continuous spectrum at $\lambda$ 3952, approximately midway between the H and K lines, show only faculæ at the limb with but very faint traces at the center of the disk. They are similar to direct photographs. These vague markings in the center gradually grow in strength until the setting is within the broad absorption band, at $\lambda$ 3962.2, where the markings appear in the center of the disk sharply defined. Careful comparisons of the markings at this wave-length with the faculæ show the agreement in form, so far as they can be compared near the limb, to be perfect. No change in form is detected until the region of $\lambda$ 3965.5 is reached. From this point on to $\lambda$ 3968.6 the change in form and size and contrast is progressive, at first slow, then rapid.

There has been some discussion as to the point in the approach to $H_2$ where we may say the faculæ are no longer depicted and the calcium flocculi appear. Mr. Evershed's[1] view confines the influence of calcium to the central $H_2$ region, or within the limits where true reversal is found, and up to this point the forms are considered faculæ. If this view is correct, then, until the $H_2$ region is reached, there should be no such change in form of the markings as appears in a series of photographs like that described above. Moreover,

[1] *The Observatory*, **27**, 164, 1904.

PLATE XVII

Slit set at

λ 3968.6

λ 3968.2

λ 3967.8

λ 3966.4

λ 3965.0

Series obtained on August 25, 1904, with slit settings approaching the center of the H line. Order: from lowest upwards.

the contrast should not increase with near approach to the $H_2$ region, for the continuous spectrum of the faculæ noticeably diminishes in intensity.

In the working hypothesis adopted by Messrs. Hale and Ellerman, the appearance of markings at the center of the disk at λ 3962.2 was considered as evidence that the calcium vapor, mixed with the general mass of condensed vapors in the faculæ had been differentiated, and its radiation alone was recorded. The agreement of the form of the markings, which they called flocculi, with the faculæ was accounted for by the intimacy of the mixture at this low level. This view may ultimately be found to be unreservedly tenable, but it would seem that the change in form of the flocculi should be progressive from this point, if we are dealing with calcium alone. As a matter of fact, pointed out above, the changes do not begin until the region λ 3965.5 is reached. It may be that this region should be considered the bounding zone of the supremacy of calcium. Two reasons, one of which has just been implied, might indicate this. It marks the change of the markings from the facular form, and from the study of the spectrum it seems to be roughly the point where the decrease begins in the intensity of the continuous spectrum of the faculæ.[1] However, a more extensive collection of plates bearing on this subject, taken at times of the very best seeing, is needed for study on this point. The taking of these occupies no unimportant place in the program of the current year. Further reference will be made to this discussion later.

Plate XVII shows a series of exposures upon a large spot-group. The settings are at λ 3965.0, λ 3966.4, λ 3967.8, λ3968.2, and λ 3968.6, and were made at the following times in the morning of August 25, 1904, $9^h\,0^m$, $9^h\,10^m$, $9^h\,12^m$, $9^h\,13^m$, and $9^h\,14^m$ C. S. T. The progressive change in form, size, and contrast is clearly brought out. Another fact clearly shown is that few if any flocculi appear in the high levels whose roots, generally in miniature, are not seen in the low levels. Even the especially brilliant points which Messrs.

[1] See also the *Astrophysical Journal* for April (**21**, 264, 1905), where Professor Hale says: "The continuous spectrum of the faculæ is usually weakened by absorption over more than half the width of the $H_1$ and $K_1$ bands," and shows Plate XIV as evidence.

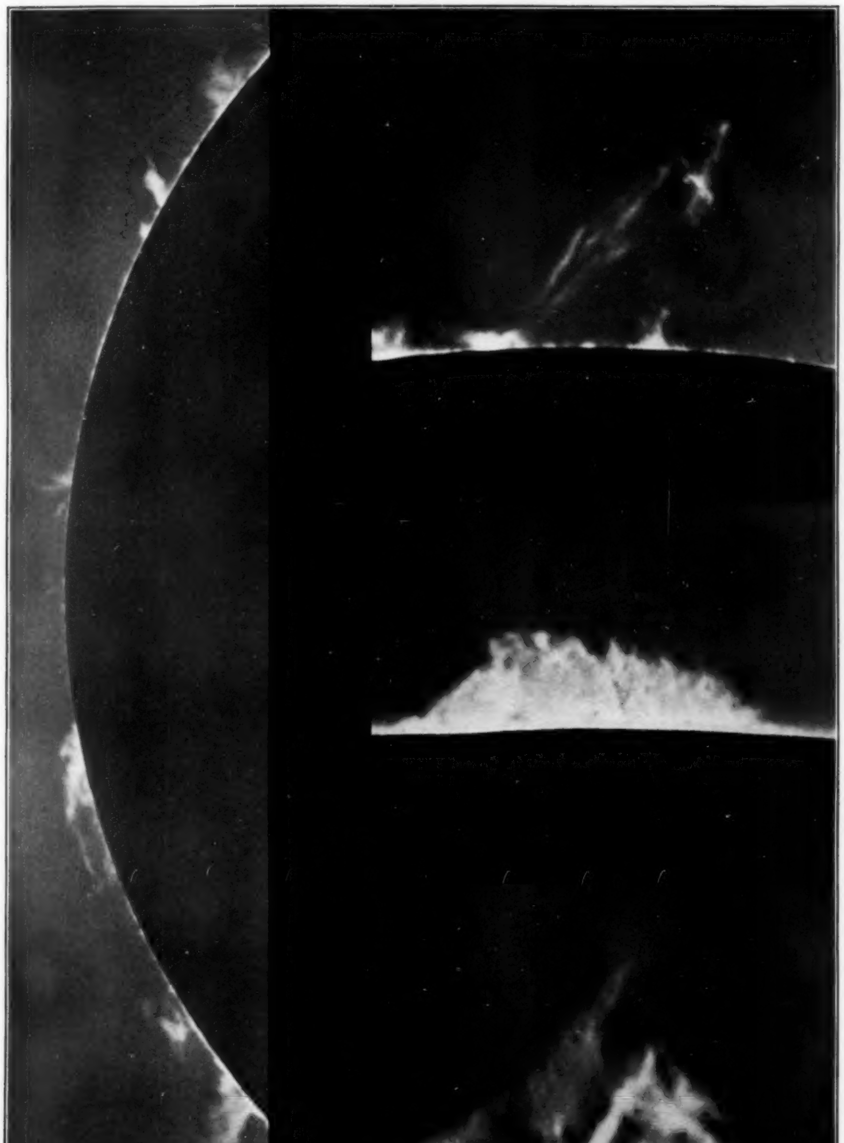
Hale and Ellerman called "eruptions" can be traced, as such, to $\lambda$ 3967. In rare cases they may be followed even farther.

Hydrogen flocculi plates, which were obtained using the lines $H_\beta$, $H_\gamma$, and $H_\delta$, in many cases show eruptions coincident with those of calcium. The excessive energy implied by these brilliant points in the flocculi is further indicated by the presence of associated prominences. In nearly all cases where these eruptions could be traced to the limb, the prominence plate revealed a prominence hovering over the flocculus. There are cases where the actual eruptive feature is seen projecting beyond the limb.

The work on the hydrogen flocculi with increased dispersion brought out few new points, but was rich in suggestions. Spectroheliograms, using the three lines mentioned above, give forms which are practically identical. Work is being continued on these lines.

The program of observations on lines other than H and K, using increased dispersion, included iron lines at $\lambda$ 4045.975 and $\lambda$ 4383.720, calcium at $\lambda$ 4226.904, strontium at $\lambda$ 4077.885, scandium at $\lambda$ 4320.907, and chromium at $\lambda$ 4254.505 and $\lambda$ 4274.958. There is difficulty in setting on some of these lines, but in many cases spectroheliograms were obtained showing detail in the center of the disk. It is a noteworthy fact that the markings near the limb differ in no wise from the faculæ, nor in the center of the disk from the forms obtained with setting at $\lambda$ 3962.2. This failure to detect a change in form may be due to one of two reasons. Perhaps the appearance of markings in the center of the disk is not indicative that we have differentiated the radiation of the element in question; but that the absorption of the general radiation of the photosphere, as we enter the shading of a line, brings out the faculæ. In dealing with a narrow line, the dispersion used may be insufficient to make the line completely fill the second slit, and therefore the radiation in question cannot be segregated to the exclusion of that of the faculæ. Or if we accept the opposite view of Messrs. Hale and Ellerman, and take the appearance of markings in the center of the disk as evidence that the element is manifesting itself, we may conclude that the dispersion is insufficient to bring out the high-level phases in the distribution of the element, or else that it does not rise to sufficient height to assume forms materially different from the

PLATE XVIII



1

2

3

4

1. Prominences on W. limb, November 3, 1904. Natural size.
2. July 20, 1904. 3. October 3, 1904. 4. August 26, 1904. Twofold enlargements.
Scale for 2, 3, and 4: 78,000 km per cm.

faculæ. Observations of Lockyer,[1] Frost,[2] and Mitchell[3] on the height which these elements reach as shown in flash spectra obtained at solar eclipses indicate low altitudes as compared to hydrogen and calcium. Very delicate setting indeed would seem to be necessary to show changes in distribution of any of these elements at different levels. It is hoped that the high dispersion which Mr. Hale is providing in the spectroheliograph at the Solar Observatory of the Carnegie Institution on Mount Wilson will be able to solve these points.

Plate XVIII shows the availability of the Rumford spectroheliograph for the photography of prominences. The year was not prolific in prominences, and no extraordinary ones were observed. Fig. 1 shows prominences obtained at 1:45 P. M. on November 3, 1904. None of the prominences are high or of unusual form, but there were few plates which showed such a wealth of them. The prominences numbered 2, 3, and 4 were photographed on July 20, 9:37 A. M.; October 3, 10:35 A. M.; and August 26, 10:01 A. M., respectively.

In closing, I wish to express my indebtedness to Mr. J. A. Brown, who as a volunteer research assistant devoted a summer to helping in these observations.

YERKES OBSERVATORY,
February 24, 1905.

[1] *Recent and Coming Eclipses*, p. 202.

[2] *Astrophysical Journal*, **12**, 307, 1900.

[3] *Ibid.*, **15**, 119, 1902.

# ON THE SPECTRA OF THE ALKALINE-EARTH FLUORIDES IN THE ELECTRIC ARC

By CH. FABRY

Most salts when placed in the electric arc give no spectrum other than that of the corresponding metal. I have discovered that the case is different for the fluorides of calcium, strontium, and barium. In analyzing the light of an electric arc taken between hollow carbons containing one of these salts we obtain, in addition to the spectrum of the metal, a very brilliant band spectrum, characteristic of the salt employed. We must thus admit the existence of vapors of these fluorides, incompletely dissociated, at the temperature of the electric arc. These spectra, which present interesting peculiarities, have seemed to me worthy of a careful investigation.

The wave-lengths have been measured, by comparison with iron lines, with the aid of a prism spectroscope recently described by M. Jobin and myself.[1] The precision of the settings is such as to permit the wave-lengths of the fine lines to be calculated to within about one part in one hundred thousand. The wave-lengths of Kayser and Runge were adopted for the iron lines.[2] The observations were wholly visual; the ultra-violet region of the three spectra was explored by photography, using a Rowland grating, without encountering a single band due to the fluorides.

I may be permitted to recall at the outset certain well-known results relating to band spectra; this is the more necessary in view of the fact that, in certain instances, the adopted terminology is not completely fixed, and consequently misunderstandings arise between different observers.

A *band* is composed in general of a large number of lines, regularly distributed in the spectrum, starting from a line which is the most brilliant of the group, and which is called the *head* of the band. Starting from the head, the intensities of the successive lines continue to decrease, while the intervals between the successive lines

[1] *Journal de Physique*, (4) **3**, 202, 1904.

[2] *Abhandlungen der K. Akad. d. Wiss.*, Berlin, 1888.

increase; we thus have a series of lines of decreasing intensity which are more and more widely separated. If each line be represented by the *frequency* of the vibratory motion, or, what amounts to the same thing, by the reciprocal of the wave-length, we find that the intervals between the successive lines increase in arithmetical progression (Deslandres).[1] It amounts to the same thing to say that the frequency $N$ of the line numbered $m$, starting from the head, may be expressed by an integral function of the second degree in $m$. In a spectroscope of small dispersion, since the lines are not separated, the appearance is that of a continuous band, sharply bounded at the *head*, whose intensity gradually decreases in the direction in which the lines extend outward from the head. The band is said to fall off toward the red or toward the violet, according to the direction in which the lines which compose the band extend outward from the head.

Ordinarily a band does not occur alone; there are several, of similar character, which usually encroach upon one another, the whole forming a *series of bands*. In this series it is especially important to consider the positions of the various heads; these, considered alone, and sharply measurable only when the dispersion is small, form a *series of heads*.

Having recalled these facts, let us return to the spectra of the fluorides, and let us consider in particular that of the fluoride of calcium, which is the easiest to obtain.[2]

Fig. 1 represents the various parts of this spectrum. Each of the strokes in the drawing is in reality a very bright line, sharply bounded on one side, and prolonged on the other side by a diffuse light, which grows fainter as the distance from the bright part increases. The appearance is exactly that of a *band* in a spectroscope of small

[1] I here pass over without mention a great number of interesting details which may be found set forth in the memoirs of M. Deslandres.

[2] There may be found in commerce, under various names (carbons for flame arcs, metallized carbons, etc.), carbons for arc lamps which contain calcium fluoride. The arc taken between these carbons is very brilliant, and gives a very bright spectrum of calcium fluoride. Some of these carbons also contain barium, and give also the spectrum of barium fluoride. The spectrum of calcium fluoride constitutes a very sensitive reaction of the fluorides: it is only necessary to impregnate carbons free from fluorides with a salt of calcium; the addition of a trace of any fluoride causes the bands of $CaFl_2$ to appear in the spectrum of the arc, particularly the green group B, which is the most brilliant.

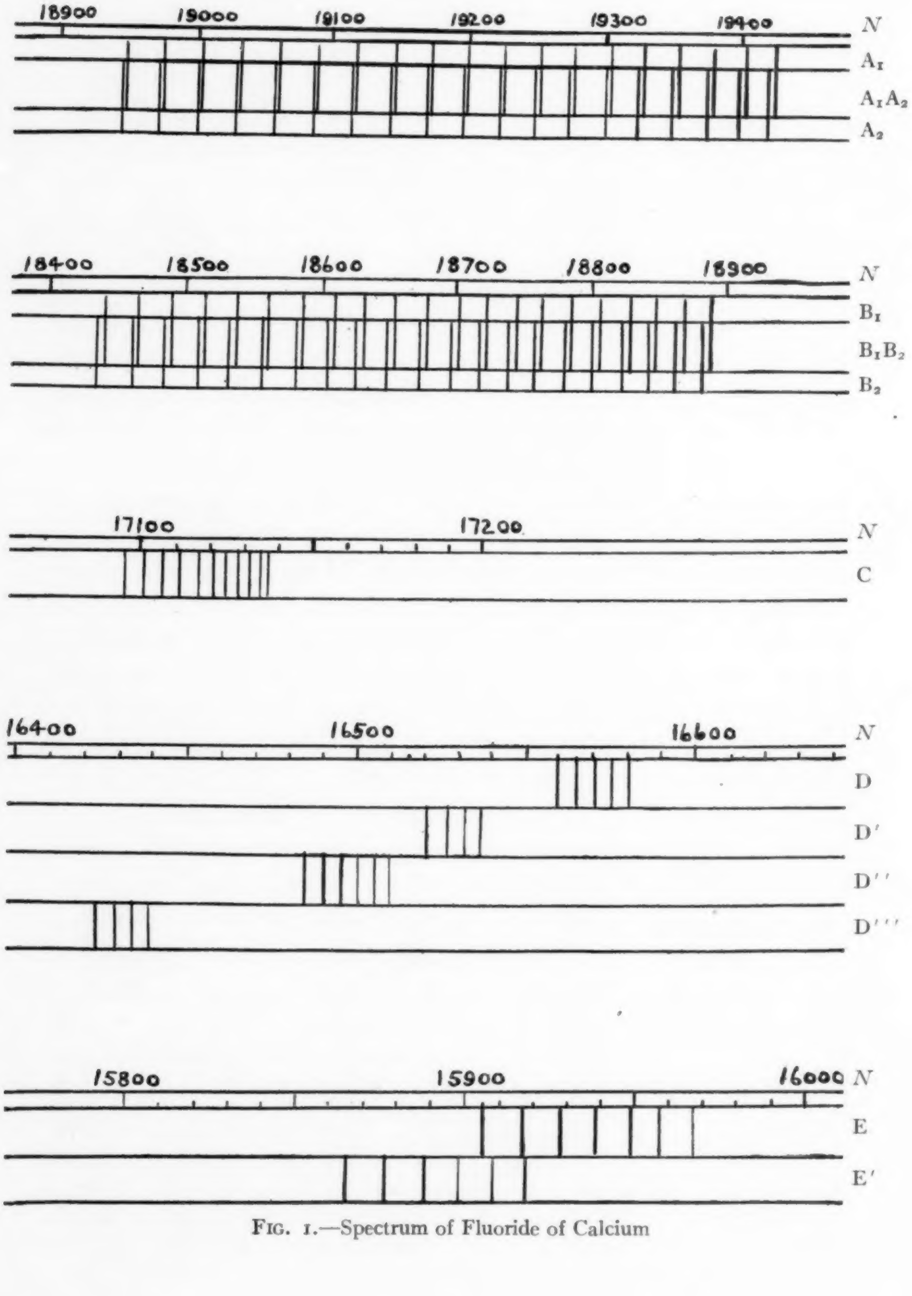



FIG. 1.—Spectrum of Fluoride of Calcium

dispersion; the bright part, sharply bounded on one side, corresponds to the head of such a band. A careful examination shows that this is precisely the interpretation of the observed appearance; the lines which constitute each band are so close together that in the neighborhood of the head I have not been able to separate them, even with the aid of a large concave Rowland grating of 7 meters radius.[1] At some distance from the head the less closely spaced lines commence to separate, but they then begin to encroach upon the lines of the neighboring heads, and measurement becomes impossible.

To sum up, each of the lines I have measured is, in reality, a *head of a band*, and the groups shown in the drawing are series of heads. These spectra cannot be used to increase our knowledge of the distribution of lines in a single band, since the lines are not separately measurable; on the other hand, their study will give us valuable information on the distribution of heads in a series, since these heads are numerous and well defined.

The various series may be divided, at a first glance, into two categories:

1. Those which are designated on the drawing by the letters A and B. From each head there extends a band falling off toward the red. The series of heads commences abruptly with the brightest head, and the others, beginning with that one, extend toward the red; the distances between the successive heads continue to increase, though slowly, in the same direction.

2. Those which are designated by C, D, and E. The corresponding bands fall off toward the violet. The series of heads also commences abruptly with the brightest head, and the following ones extend toward the violet; but the intervals between the successive heads continue to decrease.

The two kinds of series obey the following law:

If each head is represented by its *frequency* $N$,[2] we find that the intervals between the successive heads form an arithmetical progression; they follow the same law that ordinarily holds for the lines

[1] The defining power of this apparatus is, however, not much greater than that of my spectroscope, and it gives less brilliant spectra.

[2] $N$ is, in reality, a nearly constant factor, the reciprocal of the wave-length *in vacuo*. We have taken for $N$ the number of wave-lengths contained in one centimeter.

belonging to a single head, and which form a band. M. Deslandres long ago announced this law of the distribution of heads; in the present case, it is verified with a precision equal to that of the observations, in series where more than twenty heads of bands may be measured.

If, then, the heads of a certain series are numbered, the frequency $N$ of the head numbered $m$ will be expressed by a second degree function of $m$, whatever be the origin and the direction of the numbering.

There is, nevertheless, a great difference between the distribution of lines in a band and that of heads in a series. In the first case, the intensity of the lines always decreases as the interval between the lines successively augments, starting from the head of the band where this interval is very small. The same thing does not hold in the series of heads: if we take a series of the second class, the intensities continue to decrease in proportion as the intervals diminish. In a series of the first class the intensities decrease in proportion as the intervals increase, but from the first head, which is the brightest, the interval is already great, and it increases but slowly. In order to have an ana'ogous distribution of the lines in a band, it would be necessary to imagine that a considerable number of lines in the neighborhood of the head have completely disappeared.

We now come to the numerical expression of the various series. The fr quency $N$ of the head $m$ may be expressed by an equation of the form $N=A-(Bm+C)^2$. The value of the coefficien's $B$ and $C$ depends on the origin and direction of the numbering. As the series always extends only on one side from the brightest head, the most natural way of numbering has seemed to me the following:

Give to the brightest head the number 0, and to the following ones 1, 2, 3, etc. In this way $m$ will have only positive values, and the intensities will always decrease as the order numbers increase. For the series of the first class, the intervals between the successive heads increase with $m$, and $B$ is positive. It is negative for the series of the second class (the constant, $C$, being always considered positive).[1]

[1] In my first studies of the spectra of fluorides (*Comptes Rendus*, **138,** 1581, June 20, 1904) I chose a numbering such that the constant $C$ was zero, and the equation took the form $N=A-bm^2$. The initial head (the most brilliant of the series) then had a certain number $m_0$, and the following ones had numbers starting from $m_0$, in the increasing or decreasing direction, according to circumstances. I have since adopted the system of numbering just described, which is more natural and which leads to interesting relations between the numerical coefficients.

The spectrum of strontium fluoride offers striking analogies with that of the calcium salt.

The following table gives the equations of the various series of the two spectra. The corresponding series are represented by the same letters. All the values of $N$ are reciprocals of the wave-lengths *in vacuo.*

TABLE I

EQUATIONS OF THE SERIES OF HEADS OF THE BANDS IN THE SPECTRA OF THE FLUORIDES OF CALCIUM AND STRONTIUM. $N = A - (Bm + C)^2$.

(The intensities in each series decrease as the order number increases.)

| Ca $Fl_2$ | | | | Sr $Fl_2$ | | | |
|---|---|---|---|---|---|---|---|
| Series | $A$ | $B$ | $C$ | Series | $A$ | $B$ | $C$ |
| $A_1$...... | 20458.9 | +0.3776 | 32.10 | $A_1$...... | 18813.1 | +0.1835 | 32.10 |
| $A_2$...... | 20455.2 | +0.3768 | 32.10 | $A_2$...... | 18800.8 | +0.1874 | 32.10 |
| $B_1$...... | 19925.1 | +0.3209 | 32.10 | $B_1$...... | 18350.5 | +0.135 | 32.10 |
| $B_2$...... | 19918.9 | +0.31895 | 32.10 | $B_2$...... | 18327.6 | +0.128 | 32.10 |
| C....... | 17146.1 | −0.420 | 7.15 | | | | |
| D....... | 16609.3 | −0.417 | 7.07 | D....... | 15902.8 | −0.250 | 7.07 |
| D′...... | 16570.8 | −0.400 | 7.07 | D′...... | 15622.5 | −0.250 | 7.07 |
| D″...... | 16534.4 | −0.404 | 7.07 | | | | |
| D‴..... | 16473.0 | −0.383 | 7.07 | | | | |
| E....... | 16046.2 | −0.50 | 11.9 | E....... | 15492.7 | −0.34 | 11.9 |
| E′...... | 16006.5 | −0.50 | 11.9 | E′...... | 15455.9 | −0.35 | 11.9 |
| | | | | E″...... | 15161.4 | −0.35 | 11.9 |
| | | | | E‴..... | 15214.1 | −0.33 | 11.9 |

The comparison of these numbers leads to the following results:

1. If we consider the corresponding series of the two fluorides, the constant $A$ is always greater in the case of calcium than in that of strontium; in other words, the series are displaced toward greater wave-lengths when the atomic weight of the metal increases. It is well known that the same fact holds true for the lines of metals.

2. The constant $C$ has the same value for the corresponding series of the two salts. It is impossible to say whether there is an exact equality or only a partial one, since for certain series, where the number of measurable elements is small, the constants are not very well determined. Formulæ might be given involving a slight change in the constant $B$ and a corresponding small change in $C$, which would nevertheless represent the observations equally well. However, for certain series ($A$ and $B$ of calcium, $A$ of strontium)

the coefficients are very well determined and the equality of the constant $C$ is certainly true within a few hundredths of its value.

3. The constant $B$ decreases in passing, in a corresponding series, from calcium to strontium; in the latter case the lines are consequently closer together.

It is interesting to seek for analogies between the spectra which I have just described, and the band spectra previously known. The arrangement of heads which is found in the series of the second class exists in a certain number of known spectra; for example, in the beautiful bands of cyanogen. As for the structure of the series of the first class (Series $A$ and $B$), it recalls the absorption spectrum of oxygen, if the complicated part which occurs near the beginning of each group is left out of account. May this peculiar absorption spectrum be formed, in reality, not of simple bands, but of a series of heads of bands?

Barium fluoride also gives a magnificent spectrum of bands, of which only the heads are measurable; these form series of heads, as in the case of the two other fluorides. All of these series belong to the second class, i. e., the intensities of the heads diminish as the intervals between them diminish. These series are not analogous to those of the two other fluorides; they are turned in the opposite direction to the series of the second class of the fluorides of calcium and strontium. The bands which extend out from the heads fall off toward the red, and it is also in the direction of the red that the intervals between the heads increase, while their intensities decrease. The formulæ of these series are consequently of the form $N=A+(Bm+C)^2$. These bands occur in the blue; the analogues of those in the spectra of the two other fluorides should occur toward the greater wave-lengths; perhaps they are in the infra-red.

The following table gives the formulæ of the various series of barium fluoride:

TABLE II

EQUATIONS OF THE SERIES OF HEADS OF BANDS IN THE SPECTRUM OF BARIUM FLUORIDE. $N=A+(Bm+C)^2$

| Series | $A$ | $B$ | $C$ |
|---|---|---|---|
| 1 ......... | 20111.0 | −0.4302 | 9.034 |
| 2 ......... | 20197.8 | −0.441 | 7.06 |
| 3 ......... | 19842.7 | −0.4362 | 13.522 |
| 4 ......... | 19711.7 | −0.35765 | 16.715 |
| 5 ......... | 19416.2 | −0.3932 | 10.618 |
| 6 ......... | 19531.9 | −0.479 | 7.19 |

TABLE III

$CaFl_2$

| Series $A_1$ | | | Series $A_2$ | | |
|---|---|---|---|---|---|
| *m* | *N* obs. | *N* calc. | *m* | *N* obs. | *N* calc. |
| 0 | 19428.5 | 19428.5 | 0 | 19424.8 | 19424.8 |
| 1 | 403.7 | 404.0 | 1 | 400.0 | 400.4 |
| 2 | 379.8 | 379.5 | 2 | 375.9 | 375.8 |
| 3 | 354.4 | 354.5 | 3 | 351.0 | 351.0 |
| 4 | 329.4 | 329.3 | 4 | 326.3 | 325.8 |
| 5 | 303.7 | 303.7 | 5 | 300.3 | 300.3 |
| 6 | 278.0 | 277.9 | 6 | 275.1 | 274.5 |
| 7 | 252.1 | 251.9 | 7 | 248.5 | 248.4 |
| 8 | 225.6 | 225.4 | 8 | 222.6 | 222.2 |
| 9 | 198.6 | 198.9 | 9 | 196.0 | 195.6 |
| 10 | 172.0 | 171.9 | 10 | 169.3 | 168.7 |
| 11 | 144.5 | 144.5 | 11 | 141.4 | 141.3 |
| 12 | 117.5 | 117.1 | 12 | 114.8 | 114.1 |
| 13 | 089.0 | 089.3 | 13 | 085.2 | 086.3 |
| 14 | 060.4 | 060.3 | 14 | 057.4 | 057.3 |
| 15 | 032.2 | 032.8 | 15 | 029.7 | 030.0 |
| 16 | ..... | ..... | 16 | .... | ..... |
| 17 | ..... | ..... | 17 | .... | ..... |
| 18 | 18946.2 | 18945.9 | 18 | 18942.9 | 18943.3 |

| Series $B_1$ | | | Series $B_2$ | | |
|---|---|---|---|---|---|
| *m* | *N* obs. | *N* calc. | *m* | *N* obs. | *N* calc. |
| 0 | 18894.9 | 18894.7 | 0 | 18888.1 | 18888.5 |
| 1 | 873.9 | 874.0 | 1 | 867 7 | 867.9 |
| 2 | 853.0 | 853.0 | 2 | 847.3 | 847.1 |
| 3 | 831.8 | 831.9 | 3 | 826.5 | 826.2 |
| 4 | 811.0 | 810.5 | 4 | 805.1 | 805.0 |
| 5 | 789.5 | 789.1 | 5 | 783.5 | 783.5 |
| 6 | 767.2 | 767.4 | 6 | 761.9 | 761.9 |
| 7 | 745.2 | 745.5 | 7 | 740.2 | 740.1 |
| 8 | 723.1 | 723.3 | 8 | 718.6 | 718.1 |
| 9 | 701.0 | 700.9 | 9 | 696.3 | 695.9 |
| 10 | 678.2 | 678.4 | 10 | 673.4 | 673.6 |
| 11 | 655.4 | 655.6 | 11 | 651.0 | 651.0 |
| 12 | 632.1 | 632.6 | 12 | 627.6 | 628.2 |
| 13 | 609.8 | 609.4 | 13 | 605.2 | 605.2 |
| 14 | 586.5 | 586.1 | 14 | 581.7 | 581.9 |
| 15 | 561.9 | 562.5 | 15 | 559.0 | 558.5 |
| 16 | 539.0 | 538.8 | 16 | 535.1 | 534.8 |
| 17 | 515.1 | 514.7 | 17 | 511.2 | 511.0 |
| 18 | 490.6 | 490.6 | 18 | 486.6 | 487.0 |
| 19 | 465.5 | 466.1 | 19 | 461.7 | 462.7 |
| 20 | 441.8 | 441.5 | 20 | 438.0 | 438.3 |

TABLE III—*Continued*

| Series C | | | Series C | | |
|---|---|---|---|---|---|
| $m$ | $N$ obs. | $N$ calc. | $m$ | $N$ obs. | $N$ calc. |
| 0 | 17094.8 | 17095.0 | 6 | 17124.6 | 17124.7 |
| 1 | 100.6 | 100.8 | 7 | 128.3 | 128.4 |
| 2 | 106.4 | 106.3 | 8 | 131.7 | 131.7 |
| 3 | 112.2 | 111.4 | 9 | 134.6 | 134.7 |
| 4 | 116.5 | 116.2 | 10 | 137.3 | 137.4 |
| 5 | 120.8 | 120.6 | | | |

| Series D | | | Series D′ | | |
|---|---|---|---|---|---|
| $m$ | $N$ obs. | $N$ calc. | $m$ | $N$ obs. | $N$ calc. |
| 0 | 16559.2 | 16559.3 | 0 | 16520.8 | 16520.8 |
| 1 | 565.1 | 565.1 | 1 | 527.1 | 526.3 |
| 2 | 570.3 | 570.4 | 2 | 531.6 | 531.5 |
| 3 | 575.4 | 575.4 | 3 | 536.3 | 536.4 |
| 4 | 580.1 | 580.1 | | | |

| Series D″ | | | Series D‴ | | |
|---|---|---|---|---|---|
| $m$ | $N$ obs. | $N$ calc. | $m$ | $N$ obs. | $N$ calc. |
| 0 | 16484.3 | 16484.4 | 0 | 16423.0 | 16423.0 |
| 1 | 490.0 | 489.9 | 1 | 428.4 | 428.3 |
| 2 | 495.1 | 495.2 | 2 | 433.2 | 433.3 |
| 3 | 500.2 | 500.0 | 3 | 438.0 | 438.0 |
| 4 | 504.8 | 504.7 | | | |
| 5 | 508.8 | 508.9 | | | |

| Series E | | | Series E′ | | |
|---|---|---|---|---|---|
| $m$ | $N$ obs. | $N$ calc. | $m$ | $N$ obs. | $N$ calc. |
| 0 | 15904.6 | 15904.6 | 0 | 15865.2 | 15864.9 |
| 1 | 916.3 | 916.2 | 1 | 876.3 | 876.5 |
| 2 | 927.0 | 927.4 | 2 | 887.4 | 887.7 |
| 3 | 938.3 | 938.0 | 3 | 898.5 | 898.3 |
| 4 | 948.3 | 948.2 | 4 | 907.6 | 908.5 |
| 5 | 958.0 | 957.8 | 5 | 918.4 | 918.1 |
| 6 | 967.0 | 967.0 | | | |

TABLE III—*Continued*

*Sr Fl₂*

| Series $A_1$ | | | Series $A_2$ | | |
|---|---|---|---|---|---|
| *m* | *N* obs. | *N* calc. | *m* | *N* obs. | *N* calc. |
| 0 | 17782.9 | 17782.7 | 0 | 17770.3 | 17770.4 |
| 1 | 770.3 | 770.9 | 1 | ..... | ..... |
| 2 | 757.9 | 759.0 | 2 | ..... | ..... |
| 3 | 746.6 | 747.1 | 3 | ..... | ..... |
| 4 | 734.3 | 735.0 | 4 | ..... | ..... |
| 5 | 723.3 | 723.1 | 5 | ..... | ..... |
| 6 | 711.0 | 710.8 | 6 | 17697.3 | 17696.9 |
| 7 | 698.6 | 698.6 | 7 | 684.8 | 684.4 |
| 8 | 686.6 | 686.3 | 8 | 672.2 | 671.9 |
| 9 | 674.2 | 674.0 | 9 | 659.3 | 659.2 |
| 10 | 661.8 | 661.5 | 10 | 646.5 | 646.5 |
| 11 | 650.0 | 649.1 | 11 | 634.2 | 633.8 |
| 12 | 636.6 | 636.5 | 12 | 619.4 | 621.0 |
| 13 | 623.9 | 623.9 | 13 | 607.3 | 607.9 |
| 14 | 612.0 | 611.2 | 14 | 595.6 | 595.1 |
| 15 | 598.8 | 598.4 | 15 | 580.8 | 582.0 |
| 16 | 585.7 | 585.6 | 16 | 568.8 | 568.9 |
| 17 | 572.9 | 572.7 | 17 | 554.1 | 555.7 |
| 18 | 559.7 | 559.7 | 18 | 541.2 | 542.3 |
| 19 | 545.9 | 546.7 | 19 | 528.4 | 529.1 |
| 20 | 533.9 | 533.6 | 20 | 516.5 | 515.7 |
| 21 | 520.3 | 520.5 | 21 | 502.2 | 502.3 |
| 22 | 507.3 | 507.2 | 22 | 489.8 | 488.7 |
| 23 | 494.5 | 494.0 | 23 | 476.0 | 475.1 |
| 24 | 480.4 | 480.5 | 24 | 461.3 | 461.4 |
| 25 | 467.0 | 467.1 | 25 | 448.4 | 447.6 |
| 26 | 453.9 | 453.6 | | | |

| Series $B_1$ | | | Series $B_2$ | | |
|---|---|---|---|---|---|
| *m* | *N* obs. | *N* calc. | *m* | *N* obs. | *N* calc. |
| 0 | 17320.1 | 17320.1 | 0 | 17297.2 | 17297.2 |
| 1 | 311.2 | 311.5 | 1 | 289.0 | 289.0 |
| 2 | 302.8 | 302.7 | 2 | 280.9 | 280.7 |
| 3 | 293.6 | 293.9 | 3 | 272.2 | 272.4 |
| 4 | 284.6 | 285.1 | 4 | 264.6 | 264.1 |
| 5 | 276.5 | 276.3 | 5 | 255.7 | 255.7 |
| | | | 6 | 247.6 | 247.3 |
| | | | 7 | 239.1 | 238.9 |
| | | | 8 | 230.4 | 230.4 |
| | | | 9 | 222.0 | 221.9 |
| | | | 10 | 212.7 | 213.4 |
| | | | 11 | 203.9 | 205.2 |

TABLE III—*Continued*

| Series D | | | Series D′ | | |
|---|---|---|---|---|---|
| $m$ | $N$ obs. | $N$ calc. | $m$ | $N$ obs. | $N$ calc. |
| 0 | 15852.8 | 15852.8 | 0 | 15573.0 | 15572.5 |
| 1 | 856.0 | 856.3 | 1 | 576.2 | 776.0 |
| 2 | 859.6 | 859.6 | 2 | 578.5 | 579.3 |
| 3 | 862.9 | 862.9 | | | |
| 4 | 866.1 | 866.0 | | | |
| 5 | 868.9 | 868.9 | | | |
| 6 | 871.8 | 871.8 | | | |
| 7 | 874.5 | 874.5 | | | |
| 8 | 877.6 | 877.1 | | | |

| Series E | | | Series E′ | | |
|---|---|---|---|---|---|
| $m$ | $N$ obs. | $N$ calc. | $m$ | $N$ obs. | $N$ calc. |
| 0 | 15351.1 | 15351.1 | 0 | 15314.3 | 15314.3 |
| 1 | 358.4 | 359.1 | 1 | 322.6 | 322.5 |
| 2 | 366.4 | 366.8 | 3 | 330.3 | 330.5 |
| 3 | 374.3 | 374.3 | 3 | 338.0 | 338.2 |
| 4 | 383.2 | 381.6 | 4 | 346.9 | 345.7 |

| Series E″ | | | Series E‴ | | |
|---|---|---|---|---|---|
| $m$ | $N$ obs. | $N$ calc. | $m$ | $N$ obs. | $N$ calc. |
| 0 | 15019.8 | 15019.8 | 0 | 15072.4 | 15072.5 |
| 1 | 027.8 | 028.0 | 1 | 080.1 | 080.2 |
| 2 | 035.8 | 036.0 | 2 | 087.6 | 087.8 |
| 3 | 043.9 | 043.7 | 3 | 094.6 | 095.1 |
| 4 | 051.4 | 051.2 | 4 | 102.1 | 102.2 |
| | | | 5 | 110.4 | 109.0 |
| | | | 6 | 116.4 | 115.7 |

TABLE III—*Continued*

*$BaFl_2$*

| Series 1 | | |
|---|---|---|
| *m* | *N* obs. | *N* calc. |
| 0.......... | 20192.6 | 20192.6 |
| 1.......... | 185.1 | 185.0 |
| 2.......... | 177.7 | 177.8 |
| 3.......... | 170.9 | 170.9 |
| 4.......... | 165.0 | 164.5 |
| 5.......... | 158.3 | 158.4 |
| 6.......... | 152.5 | 152.6 |
| 7.......... | 147.3 | 147.3 |
| 8.......... | 142.3 | 142.3 |
| 9.......... | 137.7 | 137.6 |
| 10.......... | 133.5 | 133.4 |
| 11.......... | 129.5 | 129.5 |

| Series 2 | | |
|---|---|---|
| *m* | *N* obs. | *N* calc. |
| 0.......... | 20247.6 | 20247.6 |
| 1.......... | 241.1 | 241.6 |
| 2.......... | 235.9 | 235.9 |
| 3.......... | 231.1 | 230.7 |
| 4.......... | 225.8 | 225.8 |

| Series 3 | | |
|---|---|---|
| 0.......... | 20025.3 | 20025.6 |
| 1.......... | 014.3 | 014.0 |
| 2.......... | 003.1 | 002.7 |
| 3.......... | 19991.7 | 19991.9 |
| 4.......... | 981.2 | 981.4 |
| 5.......... | 970.9 | 971.3 |
| 6.......... | 961.7 | 961.6 |
| 7.......... | 952.2 | 952.3 |
| 8.......... | 943.1 | 943.3 |

| Series 4 | | |
|---|---|---|
| *m* | *N* obs. | *N* calc. |
| 0.......... | 19991.7 | 19991.1 |
| 1.......... | 979.2 | 979.2 |
| 2.......... | 967.4 | 967.7 |
| 3.......... | 956.0 | 955.7 |
| 4.......... | 945.1 | 945.3 |
| 5.......... | 934.3 | 934.5 |
| 6.......... | 923.9 | 923.9 |
| 7.......... | 913.7 | 913.6 |
| 8.......... | 903.2 | 903.6 |
| 9.......... | 893.6 | 893.8 |
| 10.......... | 884.1 | 884.3 |
| 11.......... | 875.0 | 875.0 |
| 12.......... | 866.4 | 866.0 |
| 13.......... | 857.2 | 857.2 |
| 14.......... | 848.8 | 848.8 |
| 15.......... | 840.1 | 840.5 |
| 16.......... | 832.8 | 832.5 |
| 17.......... | 825.3 | 824.8 |
| 18.......... | 817.7 | 817.3 |
| 19.......... | 810.4 | 810.1 |
| 20.......... | 802.9 | 803.1 |
| 21.......... | 797.1 | 796.4 |
| 22.......... | 789.6 | 790.0 |
| 23.......... | 783.9 | 783.8 |

| Series 5 | | |
|---|---|---|
| *m* | *N* obs. | *N* calc. |
| 0.......... | 19529.2 | 19528.9 |
| 1.......... | 520.7 | 520.7 |
| 2.......... | 513.1 | 512.9 |
| 3.......... | 505.6 | 505.3 |
| 4.......... | 498.7 | 498.0 |
| 5.......... | 491.3 | 491.0 |
| 6.......... | 484.5 | 484.4 |
| 7.......... | 478.4 | 478.1 |
| 8.......... | 471.7 | 472.0 |
| 9.......... | 466.2 | 466.3 |
| 10.......... | 460.9 | 460.9 |
| 11.......... | 455.6 | 455.8 |
| 12.......... | 451.1 | 451.0 |
| 13.......... | 446.3 | 446.5 |
| 14.......... | 442.6 | 442.3 |

| Series 6 | | |
|---|---|---|
| 0.......... | 19583.6 | 19583.6 |
| 1.......... | 577.1 | 576.9 |
| 2.......... | 570.7 | 570.7 |
| 3.......... | 565.1 | 565.0 |
| 4.......... | 559.7 | 559.7 |

# THE "OPTICAL POWER" OF THE ATMOSPHERE AND ITS MEASUREMENT

By KARL EXNER and W. VILLIGER

In the *Vierteljahrsschrift der Astronomischen Gesellschaft* (**37**, 3, 1902) and in the *Monthly Notices of the Royal Astronomical Society* (**53**, 40, 42, 337, 1902) Percival Lowell published communications entitled "A Standard Scale for Telescopic Observations" and "Expedition for Ascertaining the Best Location of Observatories." In these articles it is stated that the quality of the atmosphere for astronomical observations on different places on the Earth's surface is judged according to the appearance which the fixed stars in that place show in a telescope of fairly large aperture. A scale of six gradations was also made for this appearance of the stars.

In this connection it is to be noted that in the year 1887 this same idea was expressed by one of ourselves, and it was shown that it was possible not only to estimate the quality of the atmosphere, but also to measure it numerically.[1]

The action of the tremulous currents in the air is the same in observing the fixed stars through instruments of large aperture as if the eyepiece were inaccurately focused, and it cannot be remedied. We may, therefore, as in Foucault's definition of the optical power of an instrument, designate the reciprocal of the average horizontal diameter of the star's disk, estimated in seconds, at the given place as the "optical power" of the atmosphere at this place $\left(O.\,P. = \frac{1}{\text{diameter}}\right)$. This consideration gave occasion for certain quantitative studies for the measurement of Newton's phenomenon (conversion of the point image of a star into a bright surface by irregular deviation in the atmosphere), which measurements were made principally in the years 1901 and 1902 at the Royal Observatory at Munich with the $10\frac{1}{2}$-inch refractor. On making nearly three hundred measures of $D$ ($D$ being the horizontal diameter of a star's disk), it appeared:

[1] *Astronomische Nachrichten*, **116**, 321, 1887.

1. $D$ increases with $\zeta$ (true zenith distance).[1]

2. Stars of unequal magnitude, under otherwise similar circumstances, show approximately equal values of $D$.

3. The tremor (pendulum motion) of a fixed star, with sufficiently reduced aperture of a large instrument, agrees, as far as can be seen, with the diameter $D$ of the star when observed with a full-aperture instrument.

Noticing now that the $D$ of a star depends on the zenith distance, it is suggested that the measurements for the determination of the optical power should be made at limited zenith distances, perhaps at true zenith distances $\zeta = 70°$ to $80°$. If the method of the $D$-determinations is to be applied further for the determination of the optical power of a place, it is desirable to choose stars of nearly similar magnitude, as stars of the third to the sixth magnitude.

When this method of determining the diameter was applied in Munich for determining the optical power of the atmosphere at the Royal Observatory, the result in the first approximation was: $O.\ P. = \frac{1}{3}$. The details of this are given elsewhere.[2] The result $O.\ P. = \frac{1}{3}$ shows therefore that for the location of the observatory at Munich stars of the third to the sixth magnitude at 70°–80° true zenith distance show a horizontal diameter of 3″. A more detailed discussion[3] of the observations of scintillation at Munich indicates a distinct dependence of the $D$-value on the brightness of the star. At the zenith distance $\zeta = 0°$ to $50°$, the diameters of the stars measured are greater as the brightness diminishes; while in the neighborhood of the horizon, $\zeta = 60°$ to $90°$, an increase of the tremor disk is noticeable with the growing brightness of the star. The explanation of this phenomenon is carried further in the place cited. Taking account of this dependence, we have the reduction to magnitude 6.0, and zenith distance, $\zeta = 75°$, in accordance with the approximate value:

$$O.\ P.,\ \text{Munich} = \frac{1}{2.7}\,, \qquad \left\{ \begin{array}{l} m = 6.0 \\ \zeta = 75°. \end{array} \right.$$

[1] This principle was first expressed by E. R. von Oppolzer.

[2] Karl Exner and W. Villiger, "Ueber das Newton'sche Phänomen der Scintillation," *Sitzb. der kais. Akad. d. Wissensch. in Wien.*, Math.-naturw. Klasse; **111**, II*a*, 1902; and **113**, II*a*, 1904.

[3] Wiener, *Sitzungsberichte*, **113**, 1026–1037, 1904.

The determination of the optical power of an observatory can be made with very little trouble. In order to get an approximate value, it is sufficient to take four diameter measurements in the cardinal directions once each month of the year for stars of the third to the sixth magnitude at 70° to 80° true zenith distance, with an instrument of sufficiently large aperture.

The idea of optical power can also be used in a broader sense. When, for example, Secchi separated double stars of $\frac{1}{2}''$ distance in the most quiet atmosphere under the favorable conditions of Rome, while in disturbed atmosphere the diameters of the brighter stars could reach 8″; so it can be said that at a given place, at a given time and in a given direction, the optical power of the atmosphere in the first case is greater than 2, and in the second place $\frac{1}{8}$. When, further, Montigny found the greatest amplitude of the tremor of a distant object was 25″ during the day, the optical power of the intervening layer of air was $\frac{1}{50}$. Finally, when the amplitude of the tremor of the fixed stars was found by Douglas and See at the Lowell Observatory to be roughly $0\overset{''}{.}5$ to $2\overset{''}{.}0$, the optical power of the atmosphere would be *O. P.* = 1 to $\frac{1}{4}$.

# A LIST OF TWELVE STARS WHOSE RADIAL VELOCITIES VARY

By W. H. WRIGHT

The variable radial velocities of the stars on the following list have been detected while following the regular program of the D. O. Mills Expedition from the Lick Observatory, University of California, to the Southern Hemisphere. These are in addition to the five cases of variability already announced in *Lick Observatory Bulletin* No. 60.

The custom adopted at Mount Hamilton of giving values of velocities depending on approximate measurements and reductions to the nearest kilometer is followed in this paper. An exception to the general rule of giving the results of careful measurements to the nearest tenth of a kilometer is made in the case of *κ Velorum* on account of the small number of lines in its spectrum and their slightly hazy character.

Most of these determinations have been made with λ 4341 ($H\gamma$) central in the camera, using an iron comparison spectrum. A number of spectrograms have, however, been secured with λ 4450 central, using titanium for comparison purposes. These plates appear to have a systematic error of about −1.1 km (observed value—true value). Velocity determinations from such spectrograms are indicated by an asterisk (*). Values depending on poor plates are indicated by a dagger (†).

*α Phoenicis* ($\alpha = 0^h\ 21^m3$; $\delta = -42°\ 51'$)

| Date | Velocity | Measured by |
|---|---|---|
| 1903, September 15 ......... | +80.7 km | R. H. Curtiss |
| October 1 ............ | +79.0 | Palmer |
| October 5 ............ | +79.8 | R. H. Curtiss |
| 1904, August 3 ............ | +75.2 | Wright |
| September 10 ......... | +74.4* | Palmer |

*γ Phoenicis* ($\alpha = 1^h\ 24^m.0$; $\delta = -43° 50'$)

| Date | Velocity | Measured by |
|---|---|---|
| 1903, December 14.......... | +40.6 km | Palmer |
| December 22.......... | +36.4 | Palmer |
| 1904, June 22.............. | +39.0† | Palmer |
| October 3............. | +14.8* | Palmer |
| November 15.......... | +33.0* | Palmer |

The variable velocity of this star was detected by Dr. Palmer. The period as indicated by his observations appears to be roughly 190 days.

*$\theta_1$ Eridani* ($\alpha = 2^h\ 54^m.5$; $\delta = -40° 42'$)

This star is the brighter component of the telescopic double *θ Eridani*. The spectrum is a composite one of the type of that of the brighter component of *ζ Ursae Majoris*. In fact, the system of *θ Eridani* may be said to be analogous to that of *Mizar*. On the first plate secured the $H\gamma$ line, which is broad, and a number of other lines, including λ 4481, all of a similar character, were observed to be double. The magnesium line λ 4481 is the only one which can be measured with any degree of satisfaction, and even in this case settings are subject to great uncertainty. The second spectrogram showed the components of the double lines closer together, while on the third the lines are apparently single. Only one spectrogram has been secured of *$\theta_2$ Eridani*, the other component of the telescopic double. The lines on this plate are single.

| Date | Velocity | Measured by |
|---|---|---|
| 1904, December 23.......... | −65 ± +103 ± | Palmer |
| 1905, January 2............. | −30 ± +103 ± | Wright |
| January 9............ | +15 ± | Wright |

*X Eridani* ($\alpha = 4^h\ 14^m.1$; $\delta = -34° 2'$)

This spectrum belongs to the same class as that of *$\theta_1$ Eridani;* that is, both of the spectra are in evidence, though in this case the lines are quite narrow. The line λ 4481 is the only one on which measurements have been made.

| Date | Velocity | Measured by |
|---|---|---|
| 1903, October 3............ | +19 ± 1 km | Wright |
| 1904, November 10.......... | −13 ± +51 ± | Wright |
| December 7.. ......... | +20 ± | Wright |
| December 14.......... | −19 ± +70 ± | Palmer |

*δ Columbae* ($\alpha = 6^h\ 18^m.4$; $\delta = -33°\ 23'$)

| Date | Velocity | Measured by |
|---|---|---|
| 1903, December 5........... | −16.0 km | Palmer |
| 1904, February 8............ | −12.7 | Palmer |
| September 26......... | − 1.9* | Palmer |
| November 2........... | − 3.6* | Palmer |
| December 18.......... | ± 0.0 | Palmer |

The variable velocity of this star was detected by Dr. Palmer.

*A Carinae* ($\alpha = 6^h\ 47^m.6$; $\delta = -53°\ 31'$)

| Date | Velocity | Measured by |
|---|---|---|
| 1904, November 17......... | + 1.5 km* | Palmer |
| 1905, January 9............ | +28 | Wright |
| February 7........... | +48 | Wright and Palmer |

*σ Puppis* ($\alpha = 7^h\ 26^m.1$; $\delta = -43°\ 06'$)

| Date | Velocity | Measured by |
|---|---|---|
| 1904, January 15........... | +86.8 km | Palmer |
| January 29........... | +89.0 | R. H. Curtiss |
| October 25........... | +97.0* | Palmer |
| December 22.......... | +103.4 | Palmer |

The variable velocity of this star was detected by Dr. Palmer.

*a Puppis* ($\alpha = 7^h\ 48^m.8$; $\delta = -40°\ 19'$)

| Date | Velocity | Measured by |
|---|---|---|
| 1904, January 3............ | +26.5 km | Palmer |
| February 26.......... | +28 ± † | Palmer |
| November 8........... | +17.2* | Palmer |
| December 10.......... | +16.1* | Palmer |
| 1905, February 23.......... | +16 | Wright |

*α Volantis* ($\alpha=9^h\ 0^m.9$; $\delta=-66°\ 0'$)

The spectra of the two components are present, and both contain numerous lines. On only one plate is the doubling of the lines complete; but the range in the degree of sharpness of the lines on the other plates affords ample confirmation of the composite nature of the star's spectrum.

| Date | Velocity | Measured by |
|---|---|---|
| 1903, December 14.......... | +3 km (lines fairly sharp) | Wright |
| 1904, February 11.......... | +54 ± −54 ± | Wright |
| December 6.......... | +4 (lines fairly sharp) | Palmer |
| December 24.......... | +6 (lines fairly sharp) | Palmer |
| 1905, January 15.......... | +5 (lines rather hazy) | Wright |
| February 12.......... | +8 ± (lines very hazy) | Palmer |

*a Carinae* ($\alpha=9^h\ 8^m.4$; $\delta=-58°\ 33'$)

| Date | Velocity | Measured by |
|---|---|---|
| 1904, February 29.......... | + 5.5 km | Wright |
| 1905, January 30.......... | +33.2 | Palmer |
| February 9.......... | +10.0 | Wright and Palmer |
| February 22.......... | + 4.5 | Palmer |
| March 7.......... | − 1.2 | Palmer |

There is some evidence of a secondary spectrum. The $H\gamma$ line on the plate of January 30 has the appearance of a fairly narrow line displaced toward the red from the center of a rather broad absorption. It was, in fact, this peculiar appearance of the line that led me to suspect that the velocity of the star might prove variable.

*κ Velorum* ($\alpha=9^h\ 19^m.0$; $\delta=-54°\ 35'$)

| Date | Velocity | Measured by |
|---|---|---|
| 1904, March 6.......... | +67 ± km | Wright |
| 1905, January 14.......... | +13 | Wright and Palmer |
| February 20.......... | +63 ± | Wright |
| March 7.......... | +53 | Palmer |

This star has fairly narrow hydrogen and helium lines. $\lambda$ 4481 is also present and well defined. On account of the character of the star's spectrum, the values of the velocities are uncertain to the amount of a kilometer or two.

*p Velorum* ($\alpha = 10^h\ 33^m2$; $\delta = -47°\ 43'$)

This star has a composite spectrum similar to those described above, but is somewhat unique among stars of its class, from the fact that the lines, though numerous, are so sharp that settings can be made with great accuracy on the lines of both spectra.

| Date | Velocity | | Measured by |
|---|---|---|---|
| 1903, December 14.......... | +34 km | | Wright |
| 1904, February 6........... | ? | +37 | Wright |
| December 31.......... | +22 | | Wright |
| 1905, January 26........... | −10 | +40 | Wright and Palmer |

In the cases where mention is made of the fact, the detection of variable velocity has been made by Dr. Palmer, in others by the writer. In the latter cases, the plate has frequently been turned over to Dr. Palmer for more deliberate measurement than the writer could afford the time to make.

Observatory of the D. O. Mills Expedition
to the Southern Hemisphere,
Santiago de Chile, March 9, 1905.

# ON THE COMPUTATION OF THE MOON'S SPECTROGRAPHIC VELOCITY NEAR FULL MOON

By R. H. CURTISS

The determination of the Moon's spectrographic velocity from *American Ephemeris* data involves the use of the cosine of the angle ($E$) at the Earth's center between the Sun and Moon, and also the product $\sin E\frac{dE}{dt}$. $E$ and the reciprocal of its rate of change, $\frac{dt}{dE}$, are regularly tabulated in the *Nautical Almanac* except for seven or eight days at full Moon, when our satellite is often most favorably situated for spectrographic observation. As a result, the facility of computation of the Moon's radial velocity is somewhat impaired, though the problem presents no difficulty. Cos $E$ is computed directly and $\sin E\frac{dE}{dt}$ is obtained by numerical differentiation of $\cos E$, or more simply by means of differential formulæ.

A complete discussion of the five components of velocity to be considered in this case has been given by Professor Campbell.[1] Components $V_3$ and $V_4$ only are to be considered here and will be expressed invariably in kilometers per second. $V_3$ is the component of $V_2$ (the radial velocity of the Moon with reference to the Earth's center) in the line joining the Sun and Moon. $V_4$ is the component in this same line of the Moon's velocity normal to the radius vector drawn from the Moon to the Earth. The formulæ expressing these quantities are as follows:

$$V_3 = -V_2 \cos E \;,$$

$$V_4 = [4.6856]\, D_2 \sin E\frac{dE}{dt} \;,$$

where $D_2$ is the Moon's distance in kilometers from the Earth's center, and $\frac{dE}{dt}$ is expressed in seconds of arc per second of time.

Let $A$ = the Sun's right ascension ,
$\alpha$ = the Moon's right ascension ,
$D$ = the Sun's declination ,
$\delta$ = the Moon's declination .

[1] *Astrophysical Journal*, **11**, 141, 1900.

The formulæ for the computation of cos $E$ and $\sin E \frac{dE}{dt}$ are then:

$$\cos E = \sin \delta \sin D + \cos \delta \cos D \cos (A - \alpha) \ , \tag{1}$$

$$\begin{aligned} \sin E \frac{dE}{dt} = &+ \sin \delta \cos D \left[ \cos (A-\alpha) \frac{d\delta}{dt} - \frac{dD}{dt} \right] \\ &+ \cos \delta \sin D \left[ \cos (A-\alpha) \frac{dD}{dt} - \frac{d\delta}{dt} \right] \\ &+ \cos \delta \cos D \sin (A-\alpha) \frac{d(A-\alpha)}{dt} \end{aligned} \tag{2}$$

*First method.*—This involves the computation of three values of cos $E$, and a simple numerical differentiation. It is most convenient to compute $\sin E \frac{dE}{dt}$ in radians per hour by numerical differentiation of cos $E$.

Then

$$V_3 = -V_2 \cos E \ ,$$

$$V_4 = [6.4437]\, D_2 \sin E \frac{dE}{dt} = 278 \times 10^{-6} \times D_2 \sin E \frac{dE}{dt} \ .$$

Example: Determine cos $E$ and $\sin E \frac{dE}{dt}$, in radians per hour, for 1903, November $7^d\ 19^h\ 58^m$.

| November 7 | $19^h$ | $20^h$ | $21^h$ |
|---|---|---|---|
| $A$ | 222° 13′ 57″ | 222° 16′ 27″ | 222° 18′ 57″ |
| $D$ | −16 15 14 | −16 15 59 | −16 16 43 |
| $\alpha$ | 83 54 1 | 84 31 11 | 85 8 22 |
| $\delta$ | 18 16 14 | 18 17 01 | 18 17 41 |
| $A-\alpha$ | 138 19 56 | 137 45 16 | 137 10 35 |
| log sin $\delta$ | 9.49624 | 9.49655 | 9.49680 |
| sin $D$ | $9.44699_n$ | $9.44732_n$ | $9.44764_n$ |
| cos $\delta$ | 9.97753 | 9.97750 | 9.97747 |
| cos $D$ | 9.98228 | 9.98226 | 9.98223 |
| cos $(A-\alpha)$ | $9.87333_n$ | $9.86939_n$ | $9.86537_n$ |
| cos $\delta$ cos $D$ cos $(A-\alpha)$ | $9.83314_n$ | $9.82915_n$ | $9.82507_n$ |
| Add. log | 0.05264 | 0.05317 | 0.05370 |
| sin $D$ sin $\delta$ | $8.94323_n$ | $8.94387_n$ | $8.94444_n$ |
| Diff. log | 0.88991 | 0.88528 | 0.88063 |
| log cos $E$ | $9.88578_n$ | $9.88232_n$ | $9.87877_n$ |
| cos $E$ | −0.76874 | −0.76264 | −0.75643 |
| $\frac{d \cos E}{dt} = -\sin E \frac{dE}{dt}$ | +0.00610 | +0.00621 | |
| $\frac{d^2 \cos E}{dt^2}$ | | +0.00011 | |
| [$19^h\ 58^m$] cos $E$ | | −0.763 | |
| [$19^h\ 58^m$] $\sin E \frac{dE}{dt}$ | | −0.00615 | radians per hour |
| | the equivalent of | −0′.353 | per second |

*Second method.*—Both formulas (1) and (2) are employed. The hourly change in $A$ and $D$, and the velocity in $\alpha$ and $\delta$ per minute, are tabulated in the *Almanac*. It is convenient to reduce all these quantities (mentally) to seconds of arc per minute or per second of time, varying the factor in $V_4$ accordingly. All computations are accomplished with Crelle's *Rechentafeln* and three-place tables. The formulæ for $V_3$ and $V_4$, when $\frac{dE}{dt}$ is expressed in seconds of arc per second, are;

$$V_3 = -V_2 \cos E \ ,$$

$$V_4 = [4.6856]\, D_2 \sin E \frac{dE}{dt} = 485 \times 10^{-8} \times D_2 \sin E \frac{dE}{dt} \ .$$

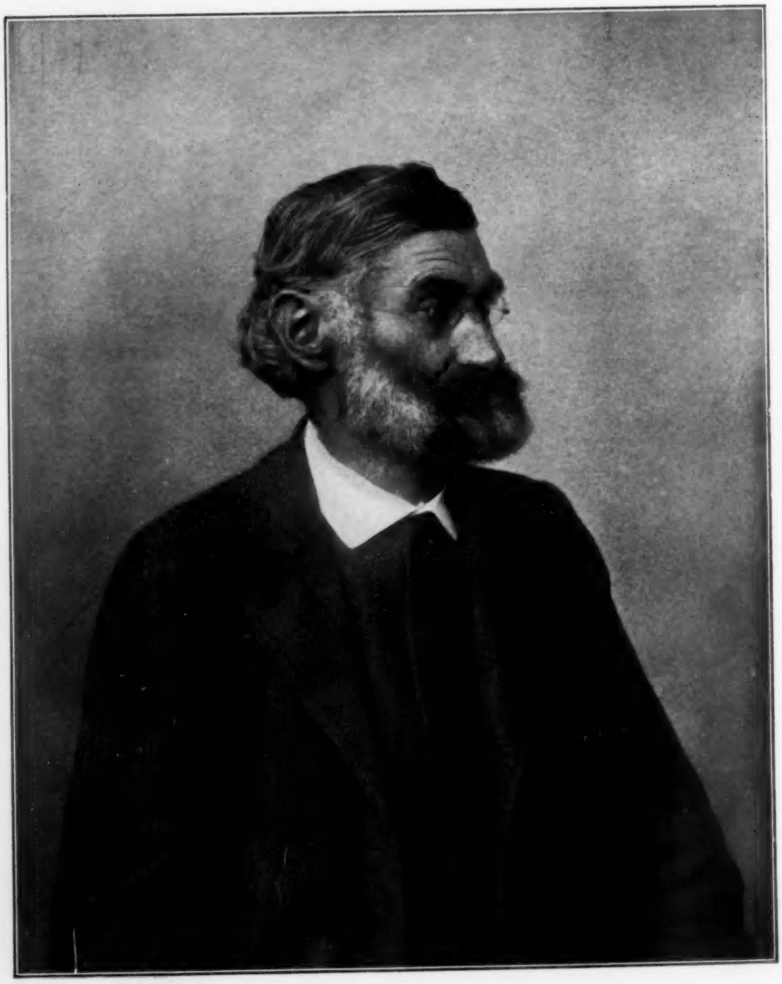
Example: Determine $\cos E$ and $\sin E \frac{dE}{dt}$ in seconds of arc per second of time.

1903, November $7^d\ 19^h\ 58^m$

| | | | |
|---|---|---|---|
| $A$ | 222° 16′ | $\frac{dA}{dt}$ per second | +0.″042 |
| $D$ | −16 16 | $\frac{d\alpha}{dt}$ per second | +0.619 |
| $\alpha$ | 84 30 | $\cos(A-\alpha)\frac{d\delta}{dt}$ | −0.009 |
| $\delta$ | 18 17 | $\frac{dD}{dt}$ per second | −0.012 |
| $A-\alpha$ | 137 46 | $\frac{d\delta}{dt}$ per second | +0.012 |
| $\sin\delta$ | +0.314 | $\cos(A-\alpha)\frac{dD}{dt}$ | +0.009 |
| $\sin D$ | −0.280 | $\cos(A-\alpha)\frac{d\delta}{dt}-\frac{dD}{dt}$ | +0.003 |
| Product | −0.088 | $\sin\delta\cos D$ | +0.3 |
| $\cos\delta$ | +0.960 | I product | +0.001 |
| $\cos D$ | +0.950 | $\cos(A-\alpha)\frac{dD}{dt}-\frac{d\delta}{dt}$ | −0.003 |
| $\cos(A-\alpha)$ | −0.740 | $\cos\delta\sin D$ | −0.3 |
| Product | −0.675 | II product | +0.001 |
| $\cos E$ | −0.763 | $(dA-d\alpha)/dt$ | −0.577 |
| | | $\sin(A-\alpha)$ | +0.672 |
| | | $\cos\delta\cos D$ | +0.912 |
| | | III product | −0.354 |
| | | $[\mathrm{I}+\mathrm{II}+\mathrm{III}]\sin E\frac{dE}{dt}$ | −0.352 per second of time |

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
March 11, 1905.

PLATE XIX



ERNST ABBE

# *Minor Contributions and Notes.*

## ERNST ABBE[1]

This distinguished authority on practical optics, to whom astronomical science is in no small measure indebted, died at Jena on January 14, after a long illness.

Born on January 23, 1840, at Eisenach, the son of an employee in a textile factory, his talents early attracted the attention of his teachers; and as a result, to a large extent through his own efforts, he was able to make his way through the university courses at Jena (1857–59) and at Göttingen (1859–61). At the latter university, after studying under Weber and Riemann, he took his degree with a thesis on the mechanical theory of heat. In 1863 he began teaching at the University of Jena as *Privat-docent*, and in 1870 was appointed *ausserordentlicher Professor*. For several years he had given assistance to Carl Zeiss, the mechanician at the Jena University, in his efforts in improving the microscope; and in 1875, at the earnest solicitation of Zeiss, Abbe became a silent partner in the firm of Zeiss & Co., the reputation of which rapidly developed. Abbe fulfilled the duties of his chair of theoretical physics and astronomy, besides those of the director of the observatory, until 1889, when at his own request, he was relieved, and thereafter gave only occasional lectures. In 1879 he entered into negotiations with Dr. Otto Schott, a practical glass-maker, with a view to the production of new kinds of glass for the requirements of practical optics. In 1882 Schott moved to Jena to devote his time to pushing more rapidly the experiments, which were first begun at Abbe's private expense. In 1884 the *Glasstechnische Laboratorium* of Schott und Genossen was established by Abbe, Schott, and Zeiss, father and son. Thereafter the financial assistance which had been given for two years by the Prussian government was no longer necessary. After the death of the senior Zeiss in 1888, and the withdrawal in the following year of his son from partnership, Abbe became the sole proprietor of the Zeiss works, but plans which he had had under way for some time finally resulted in

[1] The editors are indebted to Dr. Siegfried Czapski, managing director of the *Carl Zeiss Stiftung*, for the facts upon which this notice is based, and for the electrotype of the portrait of Dr. Abbe.

the establishment of the *Carl Zeiss Stiftung* as the sole owner of the optical works and as a partner of the glass works of Schott & Co. In 1891 Abbe transferred to the *Stiftung* all of his private property, so far as was permitted by law, and retained for himself only the position of a *Mitglied der Geschäftsleitung*, or director. Interest in his fellow-men was a passion with this university professor and successful business man, as is sufficiently evidenced by his contribution of this opportunity for large personal gain for the benefit of all his fellow-workers. An exceedingly interesting pamphlet of 145 pages, written by Professor Auerbach, of Jena, has recently been translated into English, and gives an excellent description of the great optical works, together with an explanation of the details of the numerous co-operative features of the *Stiftung*. Sociologists can find few more successful attempts at co-operation in a great common interest.

The most conspicuous scientific achievements of Abbe were: First, the development of the theory of the microscopic image of non-luminous objects. He published the elements of this theory in 1873, when it was quite contradictory to the prevailing teachings in optics; and he was constantly, though with frequent interruptions, occupied with its development. It was one of his warmest wishes, as well as that of his friends, that after he retired from the active direction of the optical works, he would find the time, so long denied, for a detailed statement of the results of his theory.

We should name as the second achievement in importance the establishment of the technics of the microscope in a rigorously scientific way upon computations involving all the elements, as radii, thicknesses, diameters, distances of lenses, and properties of the glass itself. On account of its difficulty, it was at the time hardly thought possible that this could be accomplished. The same thing had been effected by Fraunhofer for the telescope, and by Seidel and Steinheil for the photographic objective.

In the third place should be mentioned a number of remarkable optical and mechanical constructions, and numerous advances in recognizing the true nature of optical instruments. Under the one of these heads should be mentioned the Abbe refractometer, the apparatus for illuminating the microscope (1872), a system of homogeneous immersion (1878–79), the apochromatic lenses (1886), and the prism telescopes; under the other head should be enumerated the foundation of geometrical optics without reference to the means for their realization, the theory of the path of the rays, the theory of the light-power of optical instruments, and numerous contributions to the theory of errors of definition (*Abbildungsfehler*).

The full extent of the debt owed by astronomy and astrophysics to this university professor, efficient man of affairs, and conspicuous lover of his fellow-men, cannot yet be fairly realized. The microscopes, lenses, object-glasses, and prisms of the still "new Jena glass" are playing a part in the work of every active observatory; and Abbe's broad plan of establishing an impersonal institution which should call to its services the ablest talent in practical and theoretical optics will perpetuate its usefulness to science.

# *REVIEWS*

*An Introduction to the Theory of Optics.* By ARTHUR SCHUSTER. London: Edward Arnold; New York: Longmans, Green & Co., 1904. Pp. xv+340. $4.

Professor Schuster's book belongs to the broader and more modern treatises on optics of which Drude's *Lehrbuch der Optik* was the forerunner. The two books in matter and scope, speaking broadly, resemble each other, but the present one has the advantage of the four years' progress which has intervened between the two dates of publication, and by most students will be found a simpler, while in no way a less illuminating, means of approach to the broad principles which underlie this important branch of physics.

The work is divided into two parts. Part I is plainly intended as a systematic text for students entering upon the second stage of progress toward an understanding of the less involved principles of optics. Part II, which deals with some of the more general problems of radiation, has a far broader intention and is given, as necessity requires, a wholly different treatment.

Part I opens with a discussion of the kinematics and kinetics of periodic and wave-motions, followed by an introductory discussion of the nature of light and its propagation, together with a treatment of interference and diffraction, and a chapter on diffraction gratings. The theory of optical instruments precedes a discussion of the propagation of light in crystalline media and the interference of polarized light.

Part II, which will naturally prove the more interesting section to advanced students, begins with a chapter devoted to an exposition of the better-known theories of light, which is followed by a discussion of the problems of dispersion and absorption. Later chapters are upon "Rotary Effects," "The Transmission of Energy," and "The Nature of Light."

The work contains short biographical sketches of past leaders in theoretical optics, with some account of the work of each, and, what is even more important, shows the bearing of such individual contributions upon the progress of physics, thus affording an historical perspective which the student would not so easily reach for himself.

In the present volume the reader misses so full a discussion of the

problems of bodies in motion as that given by Drude; but as this whole matter in theory and experiment has not yet wholly passed out from the disputative stage, it is a compliment to the author's discretion that, for the purposes of such a book, he has held his peace upon it.

The most adequate idea which can be given of the range and purposes of the book, and the point of view of its author, is contained in the preface, from which two excerpts follow:

> There is at present no theory of optics in the sense that the elastic solid theory was accepted fifty years ago. We have abandoned that theory, and learned that the undulations of light are electromagnetic waves differing only in linear dimensions from the disturbances which are generated by oscillating electric currents or moving magnets. But so long as the character of the displacements which constitute the waves remains undefined we cannot pretend to have established a theory of light. . . . . The equations which at present represent the electromagnetic theory of light have rendered excellent service, and we must look upon them as a framework into which a more complete theory must necessarily fit; but they cannot be accepted as constituting in themselves a final theory of light.
>
> The study of physics must be based on a knowledge of mechanics, and the problem of light will only be solved when we have discovered the mechanical properties of the aether. While we are in ignorance on fundamental matters concerning the origin of electric and magnetic strains and stresses, it is necessary to introduce the theoretical study of light by a careful treatment of wave propagation through media the elastic properties of which are known. A study of the theory of sound and of the old elastic solid theory of light must precede, therefore, the introduction of the electromagnetic equation.

The reviewer feels that Professor Schuster, by clearness of exposition and the painstaking work spent in the preparation of such a timely and useful book, has put students and teachers of physics under no inconsiderable obligation.

E. F. N.

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*Astronomical Discovery*. By HERBERT HALL TURNER. London: Edward Arnold; New York: Longmans, Green & Co., 1904.

This new and very welcome book of Professor Turner's is neither a treatise nor a history, but, as explained in the preface, it is a series of half a dozen lectures upon certain important astronomical discoveries arranged "into a rough sequence according to the amount of 'chance' associated with the discovery." They are substantially the same as the course delivered at the University of Chicago in 1904, though with some changes and additions.

The subject of Chapter I is the discovery of *Uranus* and *Eros*. Chapter II deals at length with the discovery of *Neptune*, and presents some new material derived mainly from Sampson's recent memoir on the Adams MSS. Chapter III gives very fully the history of Bradley's discoveries of aberration and nutation. Chapter IV discusses some of the discoveries due to astronomical photography, and especially the remarkable phenomena presented by the "new star" of 1901. Chapter V is occupied with Schwabe's discovery of sun-spot periodicity; while, finally, Chapter VI treats of the variation of latitude, and especially of Chandler's work.

Like all of Professor Turner's writings, the book is readable and interesting; and also accurate and trustworthy, as much "readable" popular science is not. Matter which might easily become dull is enlivened by touches of humor and human interest, and by sententious bits of dry and witty wisdom. The author is especially intent to impress upon the reader how diligent labor and mere "luck" co-operate in the successful mining for scientific truth: how in the long run patient persistence in grubbing, as for instance in asteroid-hunting, often secures a rich return; while also, not infrequently, pure accident, and sudden opportunity promptly accepted, bring glorious successes.

Judged according to its scope and purpose, there is little fault to be found with the book, though many of its readers will probably confess to an Oliver Twist-like hankering for "more," and hope to have it gratified sometime in the not very distant future.

Possibly some may feel that Airy and Challis are rather hardly dealt with, especially the latter. It is, of course, quite true that if Challis had dropped everything else, and had daily reduced and compared his star-mapping work, he would have been the first to announce the new planet. But he had other pressing duties, among them a comet to be followed and observed; and comets wait for no man. Adams himself was clearly to blame for what looks like a sulky neglect to answer Airy's courteous inquiry; in fatal contrast to Leverrier's prompt reply to the same question, which led Airy to request Challis to undertake the search. Still it is not impossible that Airy's previous failure to urge the search, and his neglect to mention Adams in writing to Leverrier may have been partly due to pique at Adams' silence.

It seems, too, that the author hardly indicates how thoroughly both Adams and Leverrier were justified in assuming Bode's law as fixing the approximate distance of the hypothetical planet. First announced in 1774, the law had received brilliant confirmation in the discovery of *Uranus*, and again, twenty years later, in that of the asteroids. Any other assump-

tion would have been gratuitously unreasonable in 1845, and to work without some assumption, practically impossible.

Space does not permit more than to add that the other chapters are at least equally satisfactory, and that abundant credit is given to American astronomers, to one of whom the book is dedicated; indeed, it is possible that some of our German friends may feel that in the last chapter hardly enough credit is given to Küstner and the other observers who first authoritatively announced the variation of latitude as a fact, and organized the co-operative campaign which completely established it.

The volume is admirably printed (the only misprint we have noted is Winneche for Winnecke on p. 32), and has the crowning excellence of a good index.

C. A. Y.

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*Spectroscopic Observations of the Rotation of the Sun.* By J. Halm. Reprinted from *Transactions of the Royal Society of Edinburgh*, Vol. XLI, Part I. Edinburgh, 1904. Pp. 16.

The comparative neglect of spectroscopic investigations of the Sun's rotation since the period of Dunér's famous publication must strike the attention of students of solar physics. This is the more remarkable because the periodic character of solar disturbances, and the interesting and complicated nature of the sun-spot cycle, cannot have failed to arouse the suspicion that a variation in the period of rotation of the reversing layer might accompany the changes in the state of the Sun's activity. Under these circumstances the most probable explanation of the failure of observers to undertake the investigation seems to lie in the practical difficulty of attaching apparatus of sufficient optical power to any of the ordinary refractors.

Dr. Halm's researches, accordingly, are of great interest, not only for the results obtained, but also because of the decided advance in the general character of the apparatus used. There can be no question, quite apart from considerations of the size of the instrument which can be employed, that in investigations of such a delicate character the spectroscope should be fixed in position. In the present instance this result was attained by the use of a siderostat, which projected a beam upon a heliometer placed in a horizontal position, which, in turn, formed the solar image upon the slit of the spectroscope, itself also horizontal and stationary. As the author himself remarks, it is easy to understand how a decided increase in the accuracy of measurement was obtained under such conditions of

stability and convenience, the probable error of a single observation amounting to but one-half of that found by Dunér.

Attention should, however, be called to one defect in the apparatus which will at once be recognized by anyone who has worked in stellar spectroscopy. This is the character of the illumination of the collimating lens of the spectroscope. Not only was the lens used of too great aperture to be filled completely with light from the image-forming objective, but the nature of the latter would have prevented full illumination even had this not been the case. The beams of light from the two sections of the heliometer objective, after passing through the slit, would fall on opposite sides of the collimating lens, which would be fully illuminated by neither. This would be liable to give rise to serious error, unless the lens were focused with very great accuracy, and at the same time, the entire instrument, including both lenses and the grating, were optically perfect. Such conditions it is almost hopeless to attain, and consequently grave doubt must necessarily be thrown upon some of the numerical results obtained by Dr. Halm. In spite of its great convenience and ease of manipulation, it is difficult to see how the heliometer can be employed in a spectroscopic investigation so exacting in its requirements as that of the solar rotation.

W. S. A.

## PIETRO TACCHINI

We greatly regret to record the death on March 24, of Signor Pietro Tacchini, recently director of the *Osservatorio del Collegio Romano*, at the age of sixty-seven. Professor Tacchini has been an associate editor or collaborator of this *Journal* since its foundation, and the present development of solar physics owes much to his labors. We hope to publish in this *Journal* in due time an appropriate account of his life and works.

## NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript*, one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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